Eup

АВТОМАТИКА
И
ТЕЛЕМЕХАНИКА

Vol. 21, No. 12, December, 1960

Translation Published July, 1961

SOVIET INSTRUMENTATION AND
CONTROL TRANSLATION SERIES

Automation and Remote Control

(The Soviet Journal *Avtomatika i Telemekhanika* in English Translation)

■ This translation of a Soviet journal on automatic control is published as a service to American science and industry. It is sponsored by the Instrument Society of America under a grant in aid from the National Science Foundation, continuing a program initiated by the Massachusetts Institute of Technology.

SOVIET INSTRUMENTATION AND
CONTROL TRANSLATION SERIES

■ This translation of the Soviet Journal *Avtomatika i Telemekhanika* is published and distributed at nominal subscription rates under a grant in aid to the Instrument Society of America from the National Science Foundation. This translated journal, and others in the Series (see back cover), will enable American scientists and engineers to be informed of work in the fields of instrumentation, measurement techniques, and automatic control reported in the Soviet Union.

The original Russian articles are translated by competent technical personnel. The translations are on a cover-to-cover basis and the Instrument Society of America and its translators propose to translate faithfully all of the scientific material in *Avtomatika i Telemekhanika*, permitting readers to appraise for themselves the scope, status, and importance of the Soviet work. All views expressed in the translated material are intended to be those of the original authors and not those of the translators nor the Instrument Society of America.

Publication of *Avtomatika i Telemekhanika* in English translation started under the present auspices in April, 1958, with Russian Vol. 18, No. 1 of January, 1957. The program has been continued with the translation and printing of the 1958, 1959, and 1960 issues.

Transliteration of the names of Russian authors follows the system known as the British Standard. This system has recently achieved wide adoption in the United Kingdom, and is currently being adopted by a large number of scientific journals in the United States.

Readers are invited to submit to the Instrument Society of America comments on the quality of the translations and the content of the articles. Pertinent correspondence will be published in the Society's monthly publication, the ISA JOURNAL. Space will also be made available in the ISA JOURNAL for such replies as may be received from Russian authors to comments or questions by the readers.

1960 Volume 21 Subscription Prices:

Per year (12 issues), starting with Vol. 21, No. 1

General:	*United States and Canada*	$35.00
	Elsewhere	38.00
Libraries of nonprofit academic institutions:		
	United States and Canada	$17.50
	Elsewhere	20.50
Single issues to everyone, each		$ 6.00

1957 Volume 18, 1958 Volume 19, and 1959 Volume 20 issues also available. Prices upon request.

See back cover for combined subscription to entire Series.

Subscriptions and requests for information on back issues should be addressed to the:

Instrument Society of America
530 William Penn Place, Pittsburgh 19, Penna.

Translated and printed by Consultants Bureau Enterprises, Inc.

Automation and Remote Control

A translation of Avtomatika i Telemekhanika, a publication of the Academy of Sciences of the USSR

Volume 21, No. 12 Russian Original Dated December, 1960 July, 1961

CONTENTS

DETERMINING THE TRANSFER FUNCTIONS FOR CERTAIN SYSTEMS WITH VARYING PARAMETERS

B. E. Rudnitskii
(Leningrad)

Translated from Avtomatika i Telemekhanika, Vol. 21, No. 12, pp. 1565-1575, December, 1960
Original article submitted May 23, 1960

The paper presents a method for determining the transfer functions of systems that are described by ordinary differential equations of high order with variable coefficients, provided that the coefficients of the equations can be represented by polynomials.

INTRODUCTION

The frequency method for analyzing systems with varying parameters is based on the concept of transfer functions Y(t, p) for such systems; these depend on time t, in contrast to the transfer functions for systems with constant parameters.

We shall study a dynamic system which is described by the following n-th order differential equation with variable coefficients:

$$\sum_{k=0}^{n} a_k(t)\frac{d^k x_{\text{out}}(t)}{dt^k} = \sum_{k=0}^{m} b_k(t)\frac{d^k x_{\text{in}}(t)}{dt^k}, \quad (1)$$

where $x_{\text{in}}(t)$ and $x_{\text{out}}(t)$ are the input and output quantities of the system, respectively; $a_k(t)$ and $b_k(t)$ are time-varying coefficients.

Equation (1) can be written as follows:

$$N(t, D)\,x_{\text{out}}(t) = M(t, D)\,x_{\text{in}}(t), \quad (2)$$

where $D = d/dt$ is the differential operator, and $N(t, D)$ and $M(t, D)$ are operators for the left and right sides of equation (1), respectively. As demonstrated in [1], the transfer function for the system described by Eq. (1) is a particular solution of the differential equation

$$\sum_{k=0}^{n} \frac{1}{k!}\frac{\partial^k N(t,\ p)}{\partial p^k}\frac{\partial^k Y(t,\ p)}{\partial t^k} = M(t,\ p), \quad (3)$$

where $Y(t, p)$ is the transfer function for the system under study; the functions $N(t, p)$ and $M(t, p)$ are obtained from the operators $N(t, D)$ and $M(t, D)$, respectively, by substituting the complex quantity p for the differentiation operator D. In equation (3) the quantity t is the independent variable, and p is the parameter.

The frequency method can assure an appreciable reduction in the amount of computations compared to methods involving direct solution of the differential equations for systems with varying parameters when system stability is estimated or when the response of the system to regular or random perturbations is studied. Under these conditions, it is a very important fact that the transfer function $Y(t, p)$ of a system with varying parameters is

independent of the nature of the input perturbation. Moreover, determining the nature of the response of the system to a certain random perturbation when the frequency method is used, does not require repeated solutions for various values of the random perturbation function; this is in contrast to methods based on an empirical solution of the differential equation for the system.

In many practical cases for which other methods of analysis cause serious difficulties the frequency method makes it possible to derive the solution in convenient form (cf. below). Usually, the basic difficulties which arise on applying the frequency method to the analysis of systems with varying parameters are associated with determining the transfer function Y(t, p). The solution of Eq. (3) in general form is a problem that is no less complex than solving Eq. (2), since Eq. (3) is of the same order as (2) and its coefficients depend both on the argument t and on the parameter p. For an approximate solution of Eq. (3) for aperiodic variation of the coefficients $a_k(t)$ and $b_k(t)$ it is possible to make use of the Zadeh method [2]. Under these conditions the solution is obtained in the form of a series that converges for a sufficiently slow variation of the parameters. The basic shortcoming of this method is that it cannot be applied for a rapid variation of the parameters.

In practice, we often encounter cases where the coefficients $a_k(t)$ and $b_k(t)$ of Eq. (1) are represented by polynomials, or when these coefficients can be approximated by appropriate polynomials with a sufficient accuracy for the purposes of computation. We shall study certain possibilities for the solution of Eq. (3) if the coefficients $a_k(t)$ and $b_k(t)$ are of the form

$$a_k(t) = \sum_{l=0}^{q} a_{k,l}\, t^l, \quad b_k(t) = \sum_{l=0}^{s} b_{k,l}\, t^l.$$

In contrast to the Zadeh method, the results cited below can be applied for rapid variation of the parameters as well.

1. CERTAIN RELATIONSHIPS FOR THE TRANSFER FUNCTION OF A SYSTEM WHOSE EQUATION HAS COEFFICIENTS THAT CAN BE REPRESENTED BY POLYNOMIALS

If the coefficients $a_k(t)$ and $b_k(t)$ of Eq. (1) are represented by polynomials, then the functions N(t, p) and M(t, p) in Eq. (3) are polynomials in the variables t and p:

$$N(t, p) = \sum_{k=0}^{n} \Big(\sum_{l=0}^{q} a_{k,l}\, t^l \Big) p^k, \quad M(t, p) = \sum_{k=0}^{m} \Big(\sum_{l=0}^{s} b_{k,l}\, t^l \Big) p^k. \tag{4}$$

The following particular case of Eq. (3) will be of special interest:

$$\sum_{k=0}^{n} \frac{1}{k!} \frac{\partial^k N(t, p)}{\partial p^k} \frac{\partial^k Y(t, p)}{\partial t^k} = 1. \tag{5}$$

Theorem 1. If $Y_1(t, p)$ is a solution of Eq. (5), then the function

$$Y(t, p) = \sum_{v=0}^{s} \frac{1}{v!} \frac{\partial^v M(t, p)}{\partial t^v} \frac{\partial^v Y_1(t, p)}{\partial p^v} \tag{6}$$

is a solution of Eq. (3).

We can demonstrate the validity of the theorem by the direct substitution of (6) into (3).

Theorem 2. There exists a solution Y(t, p) of Eq. (5) which is a solution of the equation

$$\frac{1}{q!} \frac{\partial^q N(t, p)}{\partial t^q} \frac{\partial^q Y(t, p)}{\partial p^q} + \frac{1}{(q-1)!} \frac{\partial^{q-1} N(t, p)}{\partial t^{q-1}} \frac{\partial^{q-1} Y(t, p)}{\partial p^{q-1}} + \cdots$$
$$\cdots + N(t, p)\, Y(t, p) = 1. \tag{7}$$

Proof. Assume for the time being that Y(t, p) is some solution of Eq. (5). We shall formulate the function

$$u(t,\ p)=\sum_{v=0}^{q}\frac{1}{v!}\frac{\partial^{v}N(t,\ p)}{\partial t^{v}}\frac{\partial^{v}Y(t,\ p)}{\partial p^{v}}. \tag{8}$$

According to theorem 1, the function u satisfies the equation

$$\sum_{k=0}^{n}\frac{1}{k!}\frac{\partial^{k}N(t,\ p)}{\partial p^{k}}\frac{\partial^{k}u(t,\ p)}{\partial t^{k}}=N(t,\ p). \tag{9}$$

Its solution is the function u ≡ 1. The initial conditions for the function u will be the following:*

$$u(0,\ p)=1,\qquad \frac{\partial^{k}u(0,\ p)}{\partial p^{k}}=0\qquad (k=1,\ 2,\ldots,\ n-1). \tag{10}$$

We shall find the initial conditions which must be satisfied by the function Y(t, p). From relationships (10) we have

$$u(0,\ p)=\sum_{v=0}^{q}\frac{1}{v!}\frac{\partial^{v}N(0,\ p)}{\partial t^{v}}\frac{\partial^{v}Y(0,\ p)}{\partial p^{v}}\equiv 1, \tag{11}$$

$$\frac{\partial^{k}u(0,\ p)}{\partial t^{k}}=\sum_{v=0}^{q}\frac{1}{v!}\sum_{l=0}^{k}C_{k}^{l}\frac{\partial^{v+l}N(0,\ p)}{\partial t^{v+l}}\frac{\partial^{v+k-l}Y(0,\ p)}{\partial p^{v}\partial t^{k-l}}\equiv 0. \tag{12}$$

Condition (11) makes it possible to determine Y(0, p) by solving a q-th order differential equation relative to Y(0, p). Substituting Y(0, p) and its derivatives with respect to p into condition (12) for k = 1, we obtain the equation for ∂Y(0, p)/∂t. From this equation we find ∂Y(0, p)/∂t. Then we find $\partial^2 Y(0, p)/\partial t^2$ in analogous fashion from condition (12) for k = 2, etc. Thus, we obtain the initial conditions for Eq. (5). Solving this equation for these initial conditions, we obtain a function Y(t, p) which is such that the function u defined according to formula (8) will satisfy Eq. (9) for the initial conditions (10). Then u ≡ 1. Therefore, the function Y(t, p) is a solution of Eq. (7). The theorem has been proven.

Systems having equations with coefficients that are first and second degree polynomials are of special practical interest. We shall dwell in more detail on the solution of Eq. (7) for such systems.

2. THE TRANSFER FUNCTION WHEN THE COEFFICIENT N (t, p) IN EQ. (3) IS LINEAR WITH RESPECT TO THE VARIABLE t.

In that case Eq. (7) is written as

$$\frac{\partial N(t,\ p)}{\partial t}\frac{\partial Y_{1}(t,\ p)}{\partial p}+N(t,\ p)\,Y_{1}(t,\ p)=1, \tag{13}$$

where

$$N(t,\ p)=\sum_{k=0}^{n}a_{k,0}\,p^{k}+t\sum_{k=0}^{n}a_{k,1}\,p^{k}=A(p)+tB(p).$$

---

* Here and throughout $\frac{\partial^{k}u(0,\ p)}{\partial t^{k}}$ denotes $\left.\frac{\partial^{k}u(t,\ p)}{\partial t^{k}}\right|_{t=0}$. Analogously,

$$\frac{\partial^{k}Y(0,\ p)}{\partial p^{k}}=\left.\frac{\partial^{k}Y(t,\ p)}{\partial p^{k}}\right|_{t=0},\qquad \frac{\partial^{k}N(0,\ p)}{\partial t^{k}}=\left.\frac{\partial^{k}N(t,\ p)}{\partial t^{k}}\right|_{t=0}.$$

Solving Eq. (13), we obtain

$$Y_1(t, p) = \exp\left(-tp - \int \frac{A(p)}{B(p)} dp\right) \left[\int\limits_{p_0}^{p} \frac{1}{B(p)} \exp\left(tp + \int \frac{A(p)}{B(p)} dp\right) dp + C(t) \right]. \quad (14)$$

We are interested only in those solutions of $Y_1(t, p)$ which satisfy Eq. (5). This imposes a certain condition on the function $C(t)$. As shown in section I of the Appendix, the function $C(t)$ must satisfy the following equation:

$$\sum_{k=0}^{n} (a_{k,0} + a_{k,1} t) \frac{d^k C(t)}{dt^k} = e^{tp_0} \exp\left(\int \frac{A(p)}{B(p)} dp\right)\Big|_{p=p_0}. \quad (15)$$

Thus, in order for the solution (14) of Eq. (13) to satisfy Eq. (5) it is necessary and sufficient that the function $C(t)$ satisfy Eq. (15).

Remark. If the solution of Eq. (5) is found according to this theorem, then the solution of Eq. (3) will be found according to theorem 1.

3. THE TRANSFER FUNCTION FOR THE CASE WHERE THE COEFFICIENT $N(t, p)$ OF EQ. (3) IS A POLYNOMIAL OF THE SECOND DEGREE IN t.

In that case Eq. (7) is written as

$$\frac{1}{2!} \frac{\partial^2 N(t, p)}{\partial t^2} \frac{\partial^2 Y_1(t, p)}{\partial p^2} + \frac{\partial N(t, p)}{\partial t} \frac{\partial Y_1(t, p)}{\partial p} + N(t, p) Y_1(t, p) = 1, \quad (16)$$

where

$$N(t, p) = \sum_{k=0}^{n} a_{k,0} p^k + t \sum_{k=0}^{n} a_{k,1} p^k + t^2 \sum_{k=0}^{n} a_{k,2} p^k = A(p) + tB(p) + t^2 D(p).$$

We shall apply the Lemma cited in section II of the Appendix to Eq. (16). In the case under study,

$$d = d(t, p) = \frac{A(p)}{D(p)} + t \frac{B(p)}{D(p)} + t^2, \quad b = b(t, p) = \frac{B(p)}{D(p)} + 2t,$$

$$f = f(t, p) = \frac{1}{D(p)},$$

$$b^2(t, p) = \frac{B^2(p)}{D^2(p)} + 4t \frac{B(p)}{D(p)} + 4t^2, \quad \frac{db(t, p)}{dp} = \frac{\frac{dB(p)}{dp} D(p) - B(p) \frac{dD(p)}{dp}}{D^2(p)},$$

$$\varphi(t, p) = \frac{A(p)}{D(p)} + t \frac{B(p)}{D(p)} + t^2 - \frac{1}{4} \frac{B^2(p)}{D^2(p)} - t \frac{B(p)}{D(p)} - t^2 +$$

$$+ \frac{B(p) \frac{dD(p)}{dp} - D(p) \frac{dB(p)}{dp}}{2D^2(p)} =$$

$$= \frac{A(p)}{D(p)} - \frac{1}{4} \frac{B^2(p)}{D^2(p)} + \frac{B(p) \frac{dD(p)}{dp} - D(p) \frac{dB(p)}{dp}}{2D^2(p)} = \varphi(p).$$

Thus, the following postulate is valid: if in Eq. (16) the function $N(t, p)$ is a polynomial of the second degree in t, then the solution of Eq. (16) can be written as

$$Y_1(t,p)=\exp\Big(-tp-\frac{1}{2}\int\frac{B(p)}{D(p)}dp-\int g(p)\,dp\Big)\Big\{\int\limits_{p_0}^{p}\exp\Big(2\int g(p)\,dp\Big)\times$$

$$\times\Big[\int\limits_{p_0}^{p}\frac{1}{D(p)}\exp\Big(tp+\frac{1}{2}\int\frac{B(p)}{D(p)}dp-\int g(p)\,dp\Big)dp+C_1(t)\Big]dp+C_2(t)\Big\}, \tag{17}$$

where g(p) satisfies the equation

$$\frac{d}{dp}g(p)=g^2(p)+\varphi(p) \tag{18}$$

or

$$Y_1(t,p)=\gamma(p)\exp\Big(-tp-\frac{1}{2}\int\frac{B(p)}{D(p)}dp\Big)\times$$

$$\times\Big\{\int\limits_{p_0}^{p}\frac{1}{\gamma^2(p)}\Big[\int\limits_{p_0}^{p}\frac{\gamma(p)}{D(p)}\exp\Big(tp+\frac{1}{2}\int\frac{B(p)}{D(p)}dp\Big)dp+C_1(t)\Big]dp+C_2(t)\Big\}, \tag{19}$$

where $\gamma(p)$ satisfies the equation

$$\frac{d^2}{dp^2}\gamma(p)+\varphi(p)\gamma(p)=0. \tag{20}$$

As shown in section III of the Appendix, the function $C_1(t)$ in expression (17) or (19) can be chosen arbitrarily – for example; $C_1(t) = C_1$ = const; the function $C_2(t)$ satisfies the equation

$$\sum_{k=0}^{n}(a_{k,0}+a_{k,1}t+a_{k,2}t^2)\frac{d^k}{dt^k}C_2(t)=e^{tp_0}\frac{1}{F_3(p_0)}, \tag{21}$$

where

$$F_3(p)=\exp\Big(-\frac{1}{2}\int\frac{B(p)}{D(p)}dp-\int g(p)\,dp\Big)=\gamma(p)\exp\Big(-\frac{1}{2}\int\frac{B(p)}{D(p)}dp\Big).$$

Thus, in finding the transfer function from the second order equation (16) for a certain specific value of time $t = T$ it is necessary to satisfy only one initial condition which determines the value of the function $C_2(t)|_{t=T}$. This condition can usually be established on the basis of physical concepts: for example, from a study of the limits $\lim\limits_{p\to\infty} Y(t,p)$ or $\lim\limits_{p\to 0} Y(t,p)$.

The function $Y(t,p)$ which satisfies Eq. (3), is defined in terms of the function $Y_1(t,p)$ in accordance with theorem 1.

4. ON THE TRANSFER FUNCTION FOR THE CASE WHERE THE COEFFICIENT N(t, p) IN EQ. (3) IS A POLYNOMIAL OF DEGREE q IN t.

If the coefficient $N(t,p)$ in Eq. (3) is a polynomial of degree q in t, then it can be shown that the solution of Eq. (5) under these conditions can be written as

$$Y_1(t,p)=e^{-tp}F_{q+1}(p)\Big\{\int\limits_{p_0}^{p}F_q(p)\Big[\int\limits_{p_0}^{p}F_{q-1}(p)\ldots\Big(\int\limits_{p_0}^{p}e^{tp}F_1(p)\,dp+C_1(t)\Big)dp+$$

$$+C_2(t)\Big]dp+\ldots+C_{q-1}(t)\Big\}dp+C_q(t), \tag{22}$$

where the function $C_q(t)$ satisfies the equation

$$\sum_{k=0}^{n}\Big(\sum_{l=0}^{q} a_{k,l}\,t^l\Big)\frac{d^k}{dt^k}C_q(t)=\frac{e^{tp_0}}{F_{q+1}(p_0)},$$

and the functions $C_{q-1}(t), \ldots, C_1(t)$ are arbitrary.

Thus, in finding the transfer function $Y(t, p)$ from the q-th order equation (5) for a certain specific value of time $t = T$ only one initial condition besides the dependence on the order of Eq. (5) must be satisfied; this condition determines the value of the function $C_q(t)|_{t=T}$ in expression (22).

5. EXAMPLES

We shall study examples which illustrate the application of the method presented above.

Example 1. We shall determine the mean-square value $\sigma(t)$ of the signal at the output of the system shown in Fig. 1 that is subjected to a random input perturbation with the spectral density $S_{in}(\omega) = S_0$ = const for $t \gg T_{s.s.}$ [$S(\omega) = 2\int_0^\infty R_{in}(\tau)\cos\omega\tau d\tau$; $R_{in}(\tau)$ is the correlation function for the random input perturbation, $R_{in}(\tau) = S_0\delta(\tau)$, $\delta(\tau)$ is a unit pulse function, and $T_{s.s.}$ is the time required for the system to reach a steady state.

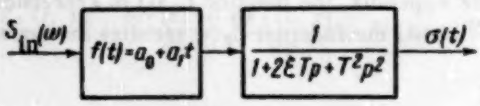

Fig. 1.

By applying theorem 1, we shall obtain the following expression for the transfer function $Y(t, p)$ of the system shown in Fig. 1:

$$Y(t, p)=\frac{a_0+a_1t}{1+2\xi Tp+T^2p^2}+a_1\frac{d}{dp}\left[\frac{1}{1+2\xi Tp+T^2p^2}\right]$$

or

$$Y(t, p)=\frac{a_0+a_1t-2\xi Ta_1+[(a_0+a_1t)\,2\xi T-2T^2a_1]\,p+(a_0+a_1t)\,T^2p^2}{(1+2\xi Tp+T^2p^2)^2}.$$

Making use of the latter expression, we find the mean-square value for the random signal at the output of the system for $t \gg T_{s.s.}$:

$$\sigma^2(t)=\frac{S_0}{2\pi}\int_{-\infty}^{\infty}|Y(t, j\omega)|^2d\omega=$$

$$=\frac{S_0}{8\xi^3T}\,[2\,(a_0+a_1t)^2\,\xi^2-(a_0+a_1t)\,a_1 2\xi T\,(1+2\xi^2)+a_1^2T^2\,(1+\xi^2+4\xi^4)].$$

Example 2. We shall find the spectral density $S_{out}(t, \omega)$ of the signal $x_{out}(t)$ at the output of the system shown in Fig. 2 for the instant time $t = a_0/a_1$, if $\xi = 0.55$, $T = 0.6$ sec, and the input signal $x_{in}(t)$ has a spectral density $S_{in}(\omega)$.

The relationship between the quantities $x_{in}(t)$ and $x_{out}(t)$ is determined from the equation

$$T^2\left(\frac{a_0}{a_1}-t\right)\frac{d^3x_{\text{out}}(t)}{dt^3}+2\xi T\left(\frac{a_0}{a_1}-t\right)\frac{d^2x_{\text{out}}(t)}{dt^2}+$$

$$+\left(\frac{a_0}{a_1}-t\right)\frac{dx_{\text{out}}(t)}{dt}+x_{\text{out}}(t)=x_{\text{in}}(t).$$

Thus, for the example under study,

$$N(t,p)=T^2\left(\frac{a_0}{a_1}-t\right)p^3+2\xi T\left(\frac{a_0}{a_1}-t\right)p^2+\left(\frac{a_0}{a_1}-t\right)p+1,$$

$$A(p)=1+\frac{a_0}{a_1}p\,(1+2\xi Tp+T^2p^2),$$

$$B(p)=-p\,(1+2\xi Tp+T^2p^2),\quad M(t,p)=1.$$

From formula (14) we obtain the following result after performing the transformations for $t=a_0/a_1$:

$$Y\left(\frac{a_0}{a_1},p\right)=1-\frac{Cp}{\sqrt{1+2\xi Tp+T^2p^2}}\exp\left(-\frac{\xi}{\sqrt{1-\xi^2}}\operatorname{arc\,tg}\frac{\xi+Tp}{\sqrt{1-\xi^2}}\right).$$

The constant C is found from the obvious condition

$$\lim_{p\to\infty}Y\left(\frac{a_0}{a_1},p\right)\to 0.$$

We obtain

$$C=T\exp\left(\frac{\pi}{2}\frac{\xi}{\sqrt{1-\xi^2}}\right).$$

Thus, we finally obtain

$$Y\left(\frac{a_0}{a_1},p\right)=1-\frac{pT\exp\left(\frac{\pi}{2}\frac{\xi}{\sqrt{1-\xi^2}}\right)}{\sqrt{1+2\xi Tp+T^2p^2}}\exp\left(-\frac{\xi}{\sqrt{1-\xi^2}}\operatorname{arc\,tg}\frac{\xi+Tp}{\sqrt{1-\xi^2}}\right)$$

and the spectral density $S_{\text{out}}(t=a_0/a_1,\ \omega)$ is determined from the expression

$$S_{\text{out}}\left(\frac{a_0}{a_1},\omega\right)=S_{\text{in}}(\omega)\left|Y\left(\frac{a_0}{a_1},j\omega\right)\right|^2.$$

Figure 3 shows the graph of the function $S_{\text{out}}(\frac{a_0}{a_1},\ \omega)\ /S_{\text{in}}(\omega)$ for the assumed values of the parameters ξ and T.

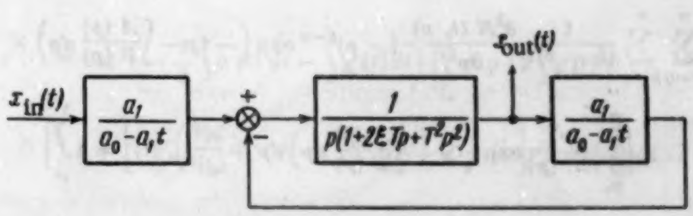

Fig. 2.

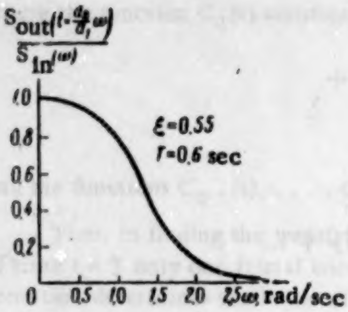

Fig. 3.

CONCLUSION

The paper has cited the relationships in the region of the variable p which determine the transfer function $Y(t, p)$ of a system with varying parameters if the coefficients in its equation can be represented by polynomials (theorems 1 and 2). These relationships make it possible to obtain the transfer functions of the systems in general form for linear variation of the coefficients in their equations, as well as in certain particular cases when the variation of the coefficients is governed by a parabolic law.

In the general case, the use of theorems 1 and 2, instead of Eq. (3) for determining the transfer function of a system with varying parameters is expedient when the condition $q < n$ is satisfied (where q is the degree of the polynomial coefficients $a_k(t)$ in the left-hand side of the equation for the system, and n is the order of the equation), since under these conditions the order of the differential equation for the transfer function is reduced.

Using the proposed method, it is possible to determine the transfer function for the series connection of inertialess sections with polynomial gain-variation laws and sections (systems) with constant parameters (the latter may have varying parameters).

The method presented for determining the transfer functions is applicable for rapid variation of the parameters when the use of the well-known Zadeh method is impossible.

The author expresses his appreciation to N. A. Lebedev for a number of valuable suggestions made during the course of writing this paper.

APPENDIX

I. We shall prove that in expression (14) the function $C(t)$ must satisfy Eq. (15). We have

$$\frac{\partial^k Y_1(t, p)}{\partial t^k} = \sum_{v=0}^{k} C_k^v (-p)^{k-v} \exp\left(-tp - \int \frac{A(p)}{B(p)}\,dp\right) \times$$
$$\times \left[\int_{p_0}^{p} \frac{p^v}{B(p)} \exp\left(tp + \int \frac{A(p)}{B(p)}\,dp\right) dp + \frac{d^v}{dt^v} C(t)\right]. \tag{23}$$

Substituting this expression into Eq. (5), we obtain

$$\sum_{k=0}^{n} \frac{1}{k!} \frac{\partial^k N(t, p)}{\partial p^k} \sum_{v=0}^{n} C_k^v (-p)^{k-v} \exp\left(-tp - \int \frac{A(p)}{B(p)}\,dp\right) \times$$
$$\times \left[\int_{p_0}^{p} \frac{p^v}{B(p)} \exp\left(tp + \int \frac{A(p)}{B(p)}\,dp\right) dp + \frac{d^v}{dt^v} C(t)\right] = 1$$

or

$$\sum_{v=0}^{n} \sum_{k=v}^{n} \frac{1}{(k-v)!} \frac{\partial^k N(t, p)}{\partial p^k} (-p)^{k-v} \exp\left(-tp - \int \frac{A(p)}{B(p)}\,dp\right) \times$$
$$\times \left[\int_{p_0}^{p} \frac{p^v}{v!\,B(p)} \exp\left(tp + \int \frac{A(p)}{B(p)}\,dp\right) dp + \frac{d^v}{dt^v} C(t)\right] = 1,$$

or

$$\sum_{v=0}^{n} \frac{\partial^v N(t, p)}{\partial p^v}\bigg|_{p=0} \exp\left(-tp - \int \frac{A(p)}{B(p)}\,dp\right) \times$$

$$\times \left[\int_{p_0}^{p} \frac{t^v}{v!\,B(p)} \exp\left(tp + \int \frac{A(p)}{B(p)}\,dp\right) dp + \frac{d^v}{dt^v} C(t)\right] = 1,$$

or

$$\int_{p_0}^{p} \frac{1}{B(p)} \exp\left(tp + \int \frac{A(p)}{B(p)}\,dp\right) \sum_{v=0}^{n} \frac{1}{v!} \frac{\partial^v N(t, p)}{\partial p^v}\bigg|_{p=0} p^v\,dp +$$

$$+ \sum_{v=0}^{n} \frac{1}{v!} \frac{\partial^v N(t, p)}{\partial p^v}\bigg|_{p=0} \frac{d^v}{dt^v} C(t) = \exp\left(tp + \int \frac{A(p)}{B(p)}\,dp\right),$$

or

$$\int_{p_0}^{p} \exp\left(tp + \int \frac{A(p)}{B(p)}\,dp\right) \frac{A(p) + tB(p)}{B(p)}\,dp +$$

$$+ \sum_{v=0}^{n} \frac{1}{v!} \frac{\partial^v N(t, p)}{\partial p^v}\bigg|_{p=0} \frac{d^v}{dt^v} C(t) = \exp\left(tp + \int \frac{A(p)}{B(p)}\,dp\right),$$

and, finally,

$$\sum_{k=0}^{n} \frac{1}{k!} \frac{\partial^k N(t, p)}{\partial p^k}\bigg|_{p=0} \frac{d^k}{dt^k} C(t) = \exp(tp_0) \exp\left(\int \frac{A(p)}{B(p)}\,dp\right)\bigg|_{p=p_0},$$

where

$$\frac{1}{k!} \frac{\partial^k N(t, p)}{\partial p^k}\bigg|_{p=0} = a_{k,0} + a_{k,1} t.$$

Thus, in expression (14) the function $C(t)$ satisfies Eq. (15).

II. It is possible to formulate the following Lemma which we shall cite without proof.

Lemma. The solution of the equation

$$\frac{d^2}{dt^2} y(t) + b(t) \frac{d}{dt} y(t) + d(t) y(t) = f(t) \tag{24}$$

can be written as

$$y(t) = \exp\left(-\frac{1}{2}\int b(t)\,dt - \int g(t)\,dt\right) \left\{\int_{t_0}^{t} \exp\left(2\int g(t)\,dt\right) \times \right.$$
$$\left. \times \left[\int_{t_0}^{t} f(t) \exp\left(\frac{1}{2}\int b(t)\,dt - \int g(t)\,dt\right) dt + C_1\right] dt + C_2\right\}, \tag{25}$$

where the function $g(t)$ satisfies the equation

$$\frac{d}{dt} g(t) = g^2(t) + \varphi(t), \tag{26}$$

and

$$\varphi(t) = d(t) - \frac{1}{4} b^2(t) - \frac{1}{2}\frac{db(t)}{dt},$$

or

$$y(t) = \gamma(t)\exp\left(-\frac{1}{2}\int b(t)\,dt\right)\times$$

$$\times\left\{\int_{t_0}^{t}\frac{1}{\gamma^2(t)}\left[\int_{t_0}^{t} f(t)\,\gamma(t)\exp\left(\frac{1}{2}\int b(t)\,dt\right)dt + C_1\right]dt + C_2\right\}, \tag{27}$$

where the function $\gamma(t)$ satisfies the equation

$$\frac{d^2}{dt^2}\gamma(t) + \varphi(t)\gamma(t) = 0. \tag{28}$$

III. We shall find the conditions which the functions $C_1(t)$ and $C_2(t)$ must satisfy in expressions (17) or (19).

We have

$$\frac{\partial^k Y_1(t,p)}{\partial t^k} = \sum_{v=0}^{n} C_k^v (-p)^{k-v} e^{-tp} F_3(p)\times$$

$$\times\left\{\int_{p_0}^{p} F_2(p)\left[\int_{p_0}^{p} p^v e^{tp} F_1(p)\,dp + \frac{d^v}{dt^v}C_1(t)\right]dp + \frac{d^v}{dt^v}C_2(t)\right\},$$

where

$$F_1(p) = \frac{\exp\left(\frac{1}{2}\int\frac{B(p)}{D(p)}dp - \int g(p)\,dp\right)}{D(p)} = \frac{\gamma(p)}{D(p)}\exp\left(\frac{1}{2}\int\frac{B(p)}{D(p)}dp\right),$$

$$F_2(p) = \exp\left(2\int g(p)\,dp\right) = \frac{1}{\gamma^2(p)},$$

$$F_3(p) = \exp\left(-\frac{1}{2}\int\frac{B(p)}{D(p)}dp - \int g(p)\,dp\right) = \gamma(p)\exp\left(-\frac{1}{2}\int\frac{B(p)}{D(p)}dp\right).$$

Substituting the latter expression into Eq. (5), we obtain

$$\sum_{k=0}^{n}\frac{1}{k!}\frac{\partial^k N(t,p)}{\partial p^k}\sum_{v=0}^{k} C_k^v(-p)^{k-v}e^{-tp}F_3(p)\times$$

$$\times\left\{\int_{p_0}^{p} F_2(p)\left[\int_{p_0}^{p} p^v e^{tp}F_1(p)\,dp + \frac{d^v}{dt^v}C_1(t)\right]dp + \frac{d^v}{dt^v}C_2(t)\right\} = 1,$$

or

$$\sum_{v=0}^{n}\sum_{k=v}^{n}\frac{1}{(k-v)!}\frac{\partial^k N(t,p)}{\partial p^k}(-p)^{k-v}e^{-tp}F_3(p)\times$$

$$\times\left\{\int_{p_0}^{p} F_2(p)\left[\int_{p_0}^{p}\frac{p^v}{v!}e^{tp}F_1(p)\,dp + \frac{d^v}{dt^v}C_1(t)\right]dp + \frac{d^v}{dt^v}C_2(t)\right\} = 1,$$

or

$$\sum_{v=0}^{n} \frac{\partial^{v} N(t, p)}{\partial p^{v}}\Big|_{p=0} e^{-tp} F_2(p) \times$$

$$\times \left\{ \int_{p_0}^{p} F_2(p) \left[\int_{p_0}^{p} \frac{p^{v}}{v!} e^{tp} F_1(p)\, dp + \frac{d^{v}}{dt^{v}} C_1(t) \right] dp + \frac{d^{v}}{dt^{v}} C_2(t) \right\} = 1.$$

or

$$\int_{p_0}^{p} F_2(p) \left[\int_{p_0}^{p} e^{tp} F_1(p) \sum_{v=0}^{n} \frac{1}{v!} \frac{\partial^{v} N(t, p)}{\partial p^{v}}\Big|_{p=0} p^{v} dp + \right.$$

$$\left. + \sum_{v=0}^{n} \frac{1}{v!} \frac{\partial^{v} N(t, p)}{\partial p^{v}}\Big|_{p=0} \frac{d^{v}}{dt^{v}} C_1(t) \right] dp + \sum_{v=0}^{n} \frac{1}{v!} \frac{\partial^{v} N(t, p)}{\partial p^{v}}\Big|_{p=0} \frac{d^{v}}{dt^{v}} C_2(t) = \frac{e^{tp}}{F_2(p)},$$

or

$$\int_{p_0}^{p} F_2(p) \left[\int_{p_0}^{p} e^{tp} F_1(p) \frac{A(p) + tB(p) + t^2 D(p)}{D(p)} dp \right] dp +$$

$$+ \int_{p_0}^{p} F_2(p)\, dp \sum_{v=0}^{n} \frac{1}{v!} \frac{\partial^{v} N(t, p)}{\partial p^{v}}\Big|_{p=0} C_1(t) +$$

$$+ \sum_{v=0}^{n} \frac{1}{v!} \frac{\partial^{v} N(t, p)}{\partial p^{v}}\Big|_{p=0} \frac{d^{v}}{dt^{v}} C_2(t) = \frac{e^{tp}}{F_2(p)},$$

or, finally,

$$\sum_{k=0}^{n} \frac{1}{k!} \frac{\partial^{k} N(t, p)}{\partial p^{k}}\Big|_{p=0} \frac{d^{k}}{dt^{k}} C_2(t) = \frac{e^{tp_0}}{F_2(p_0)},$$

where

$$\frac{1}{k!} \frac{\partial^{k} N(t, p)}{\partial p^{k}}\Big|_{p=0} = a_{k,0} + a_{k,1} t + a_{k,2} t^2.$$

Thus, we have established the following fact: if in Eq. (5) the function $N(t, p)$ is a polynomial of second degree in t, then the function $Y_1(t, p)$ defined by formulas (17) or (19) satisfies Eq. (5) for the condition that the function $C_2(t)$ satisfies Eq. (21).

In accordance with the above, the function $C_1(t)$ can be chosen arbitrarily in expressions (17) and (19); for example, $C_1(t) = C_1 = \text{const}$.

SUMMARY

There is explained a method of determination of transfer functions of systems described by common high-order differential equations with variable coefficients when these coefficients are polynomials.

LITERATURE CITED

1. A. I. Lur'e, Operational Calculus [in Russian] (Gostekhizdat, 1951).
2. L. A. Zadeh, "Frequency analysis of circuits with varying parameters", Proc. IRE, 38, No. 3 (1950).

ON METHODS FOR REALIZING A FINITE AUTOMATON WHOSE CYCLICAL NATURE IS DETERMINED BY THE VARIATION OF THE INPUT STATE

M. A. Aizerman, L. A. Gusev, L. I. Rozonoer, I. M. Smirnova, and A. A. Tal'
(Moscow)

Translated from Avtomatika i Telemekhanika, Vol. 21, No. 12, pp. 1576-1594, December, 1960
Original article submitted May 24, 1960

The paper studies three methods for an economical (in the sense of the number of states) realization of a finite automaton specified by an equation (table) and operating in a cyclic manner determined by the variations in the states of its inputs.

The methods differ as a function of the quantity of information arriving at the input of the automaton. The first method (the Huffman method) is used when the minimum information (information solely on the state of the input at the given instant) is available; the second method presupposes that additional information on the instants at which the input states change is supplied to the automaton; the third method is used when information on the state of the input at both the given instant and at a certain preceding instant is supplied to the automaton.

It is demonstrated that under conditions where the third method is used it is possible to achieve the most economical automaton.

1. A FINITE AUTOMATON AND A SEQUENTIAL MACHINE

In [1, 2] a finite automaton is defined as a dynamic system whose behavior at specified instants of time (cycles) 1, 2, . . . , p is determined by the equation

$$\varkappa(p) = F[\varkappa(p-1),\ \rho(p-1)], \tag{1}$$

where $\varkappa(p)$ and $\rho(p)$ are variables which acquire values from specified finite alphabets $\{\varkappa\}$ and $\{\rho\}$ (the alphabet $\{\varkappa\}$ contains k symbols, and the alphabet $\{\rho\}$ contains r symbols); $F(\varkappa, \rho)$ is a single-valued function.

The basic table for the automaton in [1, 2] was defined as the table (cf. Table 1) in which each column was assigned a definite value $\rho(p-1)$, and each row was assigned a definite value $\varkappa(p-1)$; the corresponding value of $\varkappa(p)$, determined on the basis of (1), is written in the square located at the intersection of any row and column.

The output of the automaton was made to correspond to the variable λ which acquired values from the alphabet $\{\lambda\}$ containing l symbols. This variable was defined by the equation

$$\lambda(p) = \Phi[\varkappa(p)], \tag{2}$$

which in [1, 2] was called the equation for the output converter.

TABLE 1

	ρ_1	ρ_2	$\cdots$	ρ_r
$\varkappa_1$	$\varkappa_3$	$\varkappa_2$	$\cdots$	$\varkappa_7$
$\varkappa_3$	$\varkappa_1$	$\varkappa_r$	$\cdots$	$\varkappa_2$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\varkappa_k$	$\varkappa_4$	$\varkappa_2$	$\cdots$	$\varkappa_2$

We shall now make two remarks concerning these concepts.

1. By introducing the new variable μ, Eq. (1) can be written as

$$\varkappa(p) = \mu(p-1), \quad \mu(p) = F[\varkappa(p), \rho(p)]. \tag{3}$$

If we eliminate μ from (3), then we immediately obtain Eq. (1). But, if we eliminate $\varkappa$ from (3), then we obtain

$$\mu(p) = F[\mu(p-1), \rho(p)]. \tag{4}$$

Therefore, it is natural to apply the term "finite automaton" to dynamic systems whose behavior is described by Eq. (4); i.e., it is applied in the general case in the system of equations (3).

2. It is natural to study output converters of a more general type

$$\lambda(p) = \Psi[\varkappa(p), \rho(p)], \tag{5}$$

instead of (2).

A converter of the type

$$\lambda(p) = \Phi[\mu(p), \rho(p)] \tag{5'}$$

can be treated as a particular case of (5), since if we take (3) into account we obtain the relationship

$$\lambda(p) = \Phi\{F[\varkappa(p), \rho(p)], \rho(p)\},$$

instead of (5'); this latter relationship does not differ from (5). Note that for the same reason a converter of the type

$$\lambda(p) = \Phi[\varkappa(p), \mu(p), \rho(p)]$$

is also no more general than the converter (5).

We shall treat Eq. (3) and (5) jointly. A dynamic system which is described by the system of equations (3) + (5) is often a sequential machine. We shall also make use of this term henceforth.*

Since Eq. (2) is a particular case of Eq. (5), the concept "sequential machine" includes the concept "finite automaton." But, on the other hand, there exists the following theorem which demonstrates that a sequential machine "cannot do" any more than a finite automaton with an output converter (cf. also thereom 1 in [4]).

Theorem. Each sequential machine M can be made to correspond to a finite automaton A with an output converter in such a way that for any initial state of machine M we find an initial state of automaton A which for any input sequence results in an output sequence of automaton A for all $p \geq 1$ that reproduces the output sequence of machine M with a lag of one cycle. Conversely, each automaton A with an output converter can be made to correspond to a sequential machine M in such a way that for any initial state of automaton A there can be found an initial state of machine M which for any input sequence will cause the output sequence of the machine M to reproduce the output sequence of automaton A with a lead of one cycle for all $p \geq 0$ (cf. proof of the theorem in Appendix I).

If the problem is to clarify what a sequential machine and a finite automaton "can do," then in view of the proven theorem cited above there is no need to analyze them separately. But these concepts are not equivalent

* In [3] the term "sequential machine" denotes a system which is described by Eq. (1) + (2); i.e., in our terminology it is a finite automaton with an output converter.

in the sense that for the performance of the same "task" a sequential machine may require a smaller number of states than the corresponding finite automaton. This fact is essential for us, since in this paper we shall study precisely the problem of minimizing the number of states.

2. STATEMENT OF THE PROBLEM

The system of equations (3) + (5) contains four variables: λ, μ, $\varkappa$, and ρ. If two of them ($\varkappa$ and μ) can be eliminated from the system in such a way that the remaining variables (λ and ρ) are related to an expression of the form

$$\lambda(p) = F_1[\lambda(p-1), \rho(p)] \tag{6}$$

or

$$\lambda(p) = F_2[\lambda(p-1), \rho(p-1)], \tag{6'}$$

then this means that the sequential machine (3) + (5) will again form an automaton* whose states are coded using the symbols $\{\lambda\}$. Of course, such an elimination of variables is far from being possible in all cases, and thus a sequential machine does not always realize a finite automaton.

In this paper we study only sequential machines which will realize a finite automaton (6); the finite automaton must be such that its operational cycles 1, 2, . . . , p are determined in a single-valued manner by the instants at which the input state changes. It is precisely with this case that we must deal when we design contact-relay networks.** It is assumed that the basic table for the automaton (6) is given. The purpose of this paper is to study various methods for realizing this basic table using a sequential machine, and for determining a minimum number of states which the sequential machine must have for various realizations when it operates in accordance with the specified table.

The methods for realizing the specified automaton are separated in accordance with the volume of information concerning the state of the input which is "communicated" to the automaton from the outside.

The minimum information on the input which must be introduced into the automaton is information on the state of the input at a specified instant (i.e., the symbol ρ(p)). When only this information is available, the realization of the automaton on the basis of delay elements (cf. for example, [1, 2]) cannot be achieved directly, since in order to control the delay elements certain additional information on the onset of a cycle must be introduced into these elements; i.e., information of the change of the input state (a signal from a "clock"***). In that case it is possible to use the method described in [2] for constructing an automaton operating in cycles that coincide with the input change from another automaton which operatores on "fast" cycles that are determined by subdividing the time axis into intervals that are small compared to the intervals ρ(p) over which the inputs change. For simplicity in presentation we shall assume that the "fast" cycling is uniform. The problem then reduces to studying the scheme shown in Fig. 1. In this scheme A (the broken line) denotes a specified automaton that operates in cycles over which the inputs are changed; F denotes a "fast" automaton operating over "fast" cycles that are not in any way associated with ρ (p); Conv. is the output converter. In other words, a specified finite auto-

*The basic table for such an automaton is also of the form shown for Table 1; however, the symbol λ is written throughout in place of $\varkappa$.

**If we have in mind a contact-relay network, then ρ denotes the state of the input contacts, and λ denotes the states of the output circuits (the actuating relays). The basic table specifies the states of the output circuits as a function of their preceding states and of their inputs; the number of automaton states (this is the quantity of interest to us) thenpredetermines the number of secondary relays which the corresponding multicycle relay network must contain. The cycling is determined by the changes of the input states if each operation occurs at the instant that the state of the input contacts of the network changes.

***It is assumed that such a "clock" can be artificial.

maton (6) is realized from a sequential machine (4) + (5) under conditions where the cycling of automaton (4) is assumed to be "fast" and in no way related to the instants ρ (p) at which the changes occur. The problem consists of determining the minimum number of states for automaton F, its basic table, and the table for the converter Conv. from the specified basic table for automaton A.

In section 3, we demonstrate how this problem is solved by the Huffman method [5].

In section 4, it is assumed that information on the instants ρ (p) at which the changes occur* is fed to the automaton from outside in addition to the symbols proper.

When such information on the input is available, automaton A can be realized directly by using delay elements. It is demonstrated that the number of states under these conditions can be made less than when realization is achieved according to the scheme shown in Fig. 1.

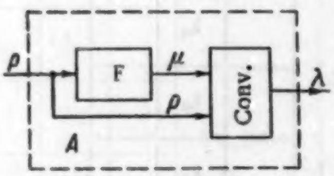

Fig. 1.

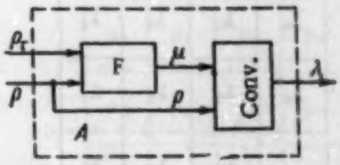

Fig. 2.

In section 5, we study the case where information on the input states ρ (p) and ρ_τ (p) at the specified instant and τ sec before, respectively, is introduced continuously into the automaton; here τ is the time interval between cycles of the "fast" automaton (Fig. 2). It is demonstrated that in this case, the number of required states for automaton F may not only be substantially reduced, but can be in a certain definite sense reduced to a limitingly small number. This means that no other measures (in particular, no further increase in the input information) can realize an automaton F with a small number of internal states.

3. THE RELIZATION OF AUTOMATON A ON THE BASIS OF USING THE STABLE STATES OF THE "FAST" AUTOMATON F

Assume that the basic table for automaton A is specified and that it is required to realize this automaton according to the scheme in Fig. 1. As a specific example of the basic table for this automaton A being synthesized, we shall study Table 2.

We shall count the number of different symbols in the alphabet $\{\lambda\}$ in some column of the table under study. We shall denote this number by a_j (the values of a_j for Table 2 are written out in its lower row) and shall determine $\alpha = \sum_{j=1}^{r} \alpha_j$ (for the case of Table 2 we have $\alpha = 10$).

Now we shall formulate a new table containing $\underline{r}$ columns (corresponding to the $\underline{r}$ inputs $\rho_1, \rho_2, \ldots, \rho_r$) and α rows. In each of the columns of the new table we write all of the different symbols λ which are encountered in the corresponding column of the original table; we write them in such a way that in each row of the new table we have written one and only one value of λ (Table 3 is filled in accordance with Table 2).

We shall study some row in the new table (for example, the fourth row from the top in Table 3). In the square of this row corresponding to input ρ_2 we have written the value λ_2. Returning to the original table (Table 2) we find the symbols located in those squares of λ, which correspond to the remaining inputs (in this particular case, ρ_1 and ρ_3). In our examples, it is λ_6 in the column ρ_1, and λ_3 in the column ρ_3. In Table 3 we write the subscripts for these λ in the fourth row which is under study; i.e., we write the number 6 in the square corresponding to ρ_1, and the number 3 in the square corresponding to ρ_3 (Table 4). Proceding in the same fashion in each row of Table 3, we fill all of its squares (Table 5).

---

*For example, there is an individual input filament which may be in one of two states; these states change only when ρ (p) changes.

TABLE 2

	ρ_1	ρ_2	ρ_3
λ_1	λ_5	λ_2	λ_3
λ_2	λ_6	λ_4	λ_3
λ_3	λ_6	λ_2	λ_6
λ_4	λ_5	λ_3	λ_3
λ_5	λ_7	λ_2	λ_1
λ_6	λ_7	λ_5	λ_1
λ_7	λ_5	λ_3	λ_6
α_j	3	4	3

TABLE 3

ρ_1	ρ_2	ρ_3
λ_6		
λ_5		
λ_7		
	λ_2	
	λ_4	
	λ_3	
	λ_5	
		λ_3
		λ_6
		λ_1

For the purpose of more efficiently isolating those squares in Table 5, in which the symbols λ_i, are written rather than the subscripts, the symbols are placed in parentheses.

The table in its new form (Table 5) represents merely a different form of notation for the original Table 2 and coincides with the "table of transfers" (the flow-table) introduced by Huffman [5].

Any basic table for a specified automaton can be reformulated in accordance with the flow table in a manner which is completely analogous to that described above using the example of Table 2 and Table 5. Further analysis leading to the solution of the problem which we have posed follows from the results of [5] and is further demonstrated using the example of Table 5.

We shall assign the corresponding symbol from the alphabet $\{\mu\}$ to each row of the flow table; we shall write it in at the left (Table 5). We shall study any column in Table 5 as the basic table for an autonomous automaton that has ten states denotes by the symbols μ_i ($i = 1, \ldots, 10$). Of these states the stable ones are those which correspond to the squares containing symbols in parentheses. The numbers in the remaining squares are treated as indications of the stable state to which the autonomous automaton is transferred during one cycle. As an example, Fig. 3 shows the graphs for the three autonomous automatons (for ρ_1, ρ_2, and ρ_3) that correspond to Table 5.

Now we shall study the automaton F which operates in arbitrary but sufficiently rapid cyclical manner* and realizes these graphs. The basic table for automaton F is immediately restored from the flow table, and, for example, for the case shown in Fig. 3 (i.e., Table 5) it has the form shown in Table 6.

Finally, we shall formulate the converter Conv. which makes each value μ_i correspond to the value λ_j which was written into the row μ_i in the flow table. For the case of Table 5 this converter is determined in accordance with Table 7.

* It is required only that the time between two cycles of automaton F be less than the time between any two changes in the value of ρ.

TABLE 4

ρ_1	ρ_2	ρ_3
λ_5		
λ_6		
λ_7		
6	λ_2	3
	λ_4	
	λ_3	
	λ_5	
		λ_3
		λ_6
		λ_1

TABLE 5

	ρ_1	ρ_2	ρ_3
μ_1	(λ_5)	2	1
μ_2	(λ_6)	5	1
μ_3	(λ_7)	3	6
μ_4	6	(λ_2)	3
μ_5	5	(λ_4)	3
μ_6	6	(λ_3)	6
μ_7	7	(λ_5)	1
μ_8	6	2	(λ_3)
μ_9	7	5	(λ_6)
μ_{10}	5	2	(λ_1)

TABLE 6

	ρ_1	ρ_2	ρ_3
μ_1	μ_1	μ_4	μ_{10}
μ_2	μ_2	μ_7	μ_{10}
μ_3	μ_3	μ_6	μ_9
μ_4	μ_2	μ_4	μ_8
μ_5	μ_1	μ_2	μ_8
μ_6	μ_2	μ_6	μ_9
μ_7	μ_3	μ_7	μ_{10}
μ_8	μ_2	μ_4	μ_8
μ_9	μ_3	μ_7	μ_9
μ_{10}	μ_1	μ_4	μ_{10}

We shall use the scheme in Fig. 4 to formulate a sequential machine from automaton F (this machine has a basic table given by Table 6) and the converter Conv. (the converter is based on Table 7).

If the readout unit reads out the symbols ρ and λ only the instants $t_1 + \tau$, $t_2 + \tau$, . . . , $t_p + \tau$, where $t_1, t_2, . . . , t_p$ are the instants at which the input changes (these instants determine the cycling) and τ is the duration of a cycle in the "fast" automaton F, then these values of ρ and λ are related to each other by the original Table 2; thus automaton A is realized.

We shall now turn from the scheme of Fig. 4, to the scheme shown in Fig. 1; i.e., we shall apply not only μ but also ρ to the input of the converter.

Using the scheme in Fig. 1, it is possible to construct not merely one but a set of sequential machines which realize the specified automaton A. They differ from one another both in terms of the automaton F and in terms of their converters Conv. Among this set of sequential machines we find minimal machines; i.e., we find those in which automaton F has the smallest number of states. We already know one sequential machine in this set (it was formulated above from the scheme in Fig. 4 and is a particular case of the scheme shown in Fig. 1); generally speaking, however, this machine is not minimal. We are required to find a minimal sequential machine.

TABLE 7

μ	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	μ_7	μ_8	μ_9	μ_{10}
λ	λ_5	λ_6	λ_7	λ_2	λ_4	λ_3	λ_5	λ_3	λ_6	λ_1

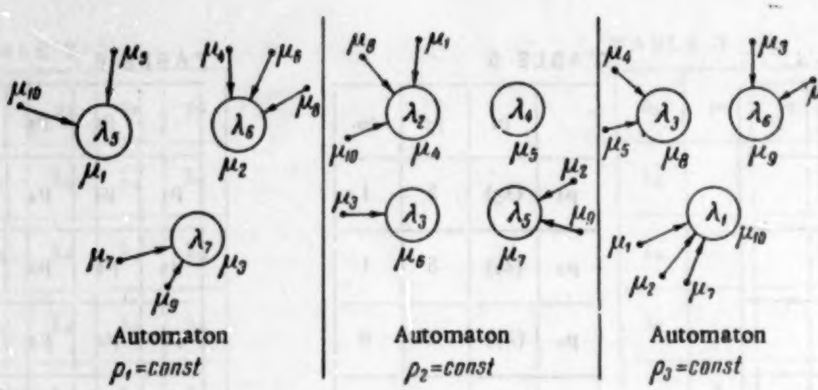

Fig. 3.

Two sequentail machines B and C shall be called equivalent (cf. [6]) if for each state b of machine B there is found at least one state c in machine C (and conversely, for each state c' of the machine C there is found a state b' of the machine B) which is such that for any identical input sequences the machines will produce identical output sequences beginning with the initial states b, c or b', c'. If the state b under study can be made to correspond to several states c, then we shall combine these states into a group $\{c\}$. It is directly evident that states combined into a group $\{c\}$ are equivalent in the sense that for any input sequence the output sequence of machine C is independent of the state in the group $\{c\}$ which is chosen as the initial state.

By studying all the states b in the machine B in sequence, we subdivide all the states for the machine C into non-intersecting groups of equivalent states.

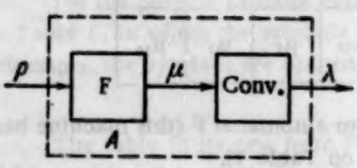

Fig. 4.

Obviously, it may occur that several states b correspond to one and the same group of states $\{c\}$. Then these states b form a group of equivalent states for machine B. If only one state b corresponds to the group of states $\{c\}$, then we shall say that this state b forms a group consisting of one element.

From the above it is evident that even though equivalent sequential machines B and C have a different number of states, these states are subdivided into an identical number of paired equivalent groups. This number of groups is an invariant; i.e., it does not change for transition from one machine to any other equivalent machine.

The invariant number can be determined from a study of any single sequential machine in the given set by combining all of its equivalent states into groups. Each group cannot contain less than one state. Therefore, it is clear that if among a set of equivalent sequential machines there were to be found a sequential machine in which each group contained only one state, then this would be the sequential machine with the minimum possible number of states in the given set.

As we have already stated, in this paper we are treating only those sequential machines which realize a finite automaton.

The states which can be established in the automaton after any change in the input are fully defined by one row in the flow table (Table 5). Assume now that the flow table contains several rows with identical numbers in identical columns.* Then it is easy to see that the states corresponding to these rows are equivalent.

In fact, due to the effect produced by any input these states make the transition into exactly the same new state; this is true because in each column of the table there cannot appear two different circles with identical symbols. Therefore, all subsequent changes of the inputs lead to one and the same sequence of changes in state; this, in turn causes the appearance of identical output sequences.

*For squares which do not contain (λ_j) the number written in the square is assumed to represent the subscript j.

Conversely, the states μ_k and μ_s are certainly not equivalent if the corresponding rows of the table contain different numbers even in one column.

Now we examine the flow table, isolate all the coincident rows, and replace each group of coincident rows with one row containing the same numbers. Under these conditions all the symbols (λ_j) which are encountered even in one of the coincident rows are retained in the single row which replaces the coincident rows. Thus, for example, in Table 5, the coincident fourth and eighth rows

μ_4	6	(λ_2)	3
μ_8	6	2	(λ_8)

are replaced by the row

μ_4	6	(λ_2)	(λ_8)

.

and the first and tenth coincident rows

μ_1	(λ_5)	2	1
μ_{10}	5	2	(λ_1)

are replaced by the row

μ_1	(λ_5)	2	(λ_1)

.

After combining all coincident rows and numbering them, we replace Table 5 with Table 8.

The flow table in which all possible substitutions of this kind are made is called the minimal flow table. Assume that it contains β rows. From the above it follows that it determines a sequential machine that is equivalent to the specified automaton and in which each group contains only one state. This is the sought for minimal sequential machine.

From the minimal flow table we immediately restore the original table for the automaton F and the converter. For the example above these correspond to Tables 9 and 10, respectively, (the dashes in Table 10 denote that the contents of the corresponding squares have no significance for the operation of the sequential machine).

Automaton F can now be formulated by the methods of aggregation from any appropriate set of elements. When the relay method of realization is used, we require at least s_0 secondary relays, where s_0 is the minimum number satisfying the inequality

$$2^{s_0} \geq \beta.$$

In order to avoid "competition" between elements, the number of relays must be increased. When the $(2s_0 + 1)$-realization developed by Huffman [5] is used for this purpose we require $2s_0 + 1$ relays.

4. THE REALIZATION OF AUTOMATON $\underline{A}$ USING DELAY ELEMENTS

Assume now that in addition to the information on the instantaneous value ρ of the automaton input we have a special device ("a clock") which signals the onset of a cycle of automaton A. In this case, for which the cyclical operation is determined by any change in the input state, this means that we have additional information available on the fact that the symbol ρ changed at the input of the automaton.

When such additional information is present there is no longer any need to construct automaton A (whose cyclical operation is determined by the instants at which the input states change) using the "fast" automaton F. In that case automaton A can be constructed directly from elements operating in the required cyclical manner; this can be done using aggregation methods for constructing automaton A from any complete set in the manner described in [1]. If, for example, the automaton is constructed from a complete set consisting of delay elements and logic converters, then the additional information on the change on the input is applied to all the delay elements. In realizing the automaton according to the method described in [1] it will have l states, where l is the number of rows in the basic table being realized (in our example this is Table 2). Sometimes $l < \beta$, and then such a construction of automaton A is more economical than that described in section 2 involving its construction from a

"fast" automaton F. But, it is also possible that $l > \beta$; in these cases the realization of the automaton described in [1] is less economical than its realization from automaton F.

Note, however, that in [1] no limitations were imposed on the sequences $\rho(t)$ which could be applied to the input of the automaton.

In the case under study here, for which the cyclical operation is determined solely by the instants at which the symbol ρ changes, the choice of the input sequences $\rho(t)$ is limited by the fact that one and the same symbol cannot be repeated twice in succession in the input sequence. Thus, for example, the sequence

Cycle	1	2	3	4	5	...
ρ	ρ_1	ρ_7	ρ_3	ρ_6	ρ_6	...

cannot occur, since the fifth cycle cannot arise as long as the symbol ρ_6 is not replaced by another symbol.

The limitation which this condition imposes on the possible input sequence makes it possible to use elements operating in the required cyclical manner to construct an automaton A which realizes the specified basic table and has less than l states. Below we shall demonstrate that for such a realization we require no more than β (and often substantially less) states.

We shall use R to denote the set of all input sequences $\rho(t)$ which satisfy the limitation indicated above. Now the problem can be formulated as follows. Assume that we have specified the basic table for automaton A with states and inputs taken from the alphabets $\{\lambda\}$ and $\{\rho\}$, respectively. It is required to construct a sequential machine B,

$$\begin{gathered} \varkappa(p) = \mu(p-1), \\ \mu(p) = F[\varkappa(p), \rho(p)], \\ \lambda(p) = \Phi[\varkappa(p), \rho(p)], \end{gathered} \tag{7}$$

such that for all sequences belonging to the set R it will realize the table for automaton A and have the minimum possible number of states $\underline{s}$. In other words, it is required to find the minimal sequential machine from the class of sequential machines which are equivalent relative to the set R and realize the basic table of the automaton.*

In order to solve this problem we shall use the specified basic tables of the realized automaton A to formulate the corresponding flow table (for the example under study this is Table 5); then we modify this flow table in the following manner.

1. In all squares containing just numbers [and not symbols (λ_j)] we write the additional symbol μ indicating the state into which the system transfers due to the flow table. Thus, for example, in row μ_4 and column ρ_1 we write the additional symbol μ_2 along with the number 6, since in this column the symbol λ_6 in parentheses is located in row μ_2. As a result we obtain Table 11 instead of Table 5.

2. After that we eliminate all symbols corresponding to stable states from the flow table; i.e., we eliminate all the symbols λ_i in parentheses. The squares in which these symbols were written are left empty.

3. In the entire table we replace the symbols μ by the symbols $\varkappa$ while retaining the subscripts. As a result of this we obtain Table 12 instead of Table 5.

In contrast to the flow table which describes the operation of the machine

* Here we shall call two sequential machines B and C equivalent relative to the set of input sequences R if each state of machine B can be made to correspond to a state of machine C (and conversely); thus, when any two corresponding states are assumed to be the initial states, machines B and C respond to any identical input sequences belonging to the set R by producing identical output sequences $\lambda(t)$ (cf. [7]).

TABLE 8

	ρ_1	ρ_2	ρ_3
μ_1	(λ_5)	2	(λ_1)
μ_2	(λ_6)	5	1
μ_3	(λ_7)	3	6
μ_4	6	(λ_2)	(λ_3)
μ_5	5	(λ_4)	3
μ_6	6	(λ_3)	6
μ_7	7	(λ_5)	1
μ_8	7	5	(λ_6)

TABLE 9

	ρ_1	ρ_2	ρ_3
μ_1	μ_1	μ_4	μ_1
μ_2	μ_2	μ_7	μ_1
μ_3	μ_3	μ_6	μ_8
μ_4	μ_2	μ_4	μ_4
μ_5	μ_1	μ_5	μ_4
μ_6	μ_2	μ_6	μ_8
μ_7	μ_3	μ_7	μ_1
μ_8	μ_3	μ_7	μ_8

TABLE 10

	ρ_1	ρ_2	ρ_3
μ_1	λ_5	—	λ_1
μ_2	λ_6	—	—
μ_3	λ_7	—	—
μ_4	—	λ_2	λ_3
μ_5	—	λ_4	—
μ_6	—	λ_3	—
μ_7	—	λ_5	—
μ_8	—	—	λ_6

$$\mu(p) = F[\mu(p-1),\ \rho(p)], \qquad \lambda(p) = \Phi[\mu(p),\ \rho(p)],$$

the reformulated Table 12 describes the operation of the machine

$$\varkappa(p) = F[\varkappa(p-1),\ \rho(p-1)], \qquad \lambda(p) = \Phi[\varkappa(p),\ \rho(p)].$$

In view of the reformulated flow table the states of the machine can be subdivided into non-intersecting groups corresponding to the symbols $\varkappa_i$; these groups may appear for one another input symbol ρ_j. Thus, for example, according to Table 12 the input ρ_1 corresponds to the group of states $G_1\{\varkappa_1, \varkappa_2, \varkappa_3\}$, the input ρ_2 corresponds to the group $G_2\{\varkappa_4, \varkappa_5, \varkappa_6, \varkappa_7\}$, and the input ρ_3 corresponds to the group $G_3\{\varkappa_8, \varkappa_9, \varkappa_{10}\}$. But, in view of the limitation imposed on the class of possible input sequences, two identical input symbols cannot be encountered in succession. Therefore, if the machine is in one of the states in group G_i, then the symbol ρ_i cannot appear at the input.

It is precisely for this reason that the squares corresponding to states in the group G_i are left unfilled in the column ρ_i of the reformulated flow table. In determining the sequence $\varkappa(t)$ from the specified sequence $\rho(t)$ it is never necessary to know the contents of those squares, provided only that the input sequence of symbols ρ belongs to the set R (i.e., if it does not contain repeating symbols). Therefore, these empty squares can be filled quite arbitrarily.

The states of the machine can be called equivalent relative to the set of input sequences R if a machine having these states as initial states responds with identical output sequences to identical input sequences in the set R.

The problem of constructing a minimal machine reduces to the problem of combining states that are equivalent relative to R. However, the problem of matching states in machines for which the set of input sequences is bounded involves essential peculiarities in comparison to the analogous problem in the preceding section (cf. [7]). We shall study any two rows in Table 12 and compare them; we shall pay attention only to the output symbols and shall take into account the fact that we can write an arbitrary symbol in the empty squares. We shall say that two rows coincide if we can fill the empty squares in such a way that the squares in identical columns contain identical symbols $\varkappa$.

TABLE 11

	ρ_1	ρ_2	ρ_3
μ_1	(λ_5)	$\mu_4, 2$	$\mu_{10}, 1$
μ_2	(λ_6)	$\mu_7, 5$	$\mu_{10}, 1$
μ_3	(λ_7)	$\mu_8, 3$	$\mu_9, 6$
μ_4	$\mu_2, 6$	(λ_2)	$\mu_8, 3$
μ_5	$\mu_1, 5$	(λ_4)	$\mu_8, 3$
μ_6	$\mu_2, 6$	(λ_3)	$\mu_9, 6$
μ_7	$\mu_3, 7$	(λ_5)	$\mu_{10}, 1$
μ_8	$\mu_2, 6$	$\mu_4, 2$	(λ_3)
μ_9	$\mu_3, 7$	$\mu_7, 5$	(λ_6)
μ_{10}	$\mu_1, 5$	$\mu_4, 2$	(λ_1)

TABLE 12

	ρ_1	ρ_2	ρ_3
$\varkappa_1$		$\varkappa_4, 2$	$\varkappa_{10}, 1$
$\varkappa_2$		$\varkappa_7, 5$	$\varkappa_{10}, 1$
$\varkappa_3$		$\varkappa_8, 3$	$\varkappa_9, 6$
$\varkappa_4$	$\varkappa_2, 6$		$\varkappa_8, 3$
$\varkappa_5$	$\varkappa_1, 5$		$\varkappa_8, 3$
$\varkappa_6$	$\varkappa_3, 6$		$\varkappa_9, 6$
$\varkappa_7$	$\varkappa_3, 7$		$\varkappa_{10}, 1$
$\varkappa_8$	$\varkappa_2, 6$	$\varkappa_4, 2$	
$\varkappa_9$	$\varkappa_3, 7$	$\varkappa_7, 5$	
$\varkappa_{10}$	$\varkappa_1, 5$	$\varkappa_4, 2$	

If two rows coincide, then it follows immediately from the definition of "states equivalent relative to R" that the symbols $\varkappa$ written in these rows corresponds to non-equivalent states. Such rows cannot be made to coincide. Examples of such rows are 1 and 3, 4 and 10, etc., in Table 12.

However, if two rows coincide, then we obtain identical output subscripts in each column if the empty squares are appropriately filled. In view of the definition, the symbols $\varkappa$ used in these rows correspond to states that are equivalent to R.

Thus, in Table 12, for example, such states are $\varkappa_1$ and $\varkappa_{10}$ (the first and tenth rows, respectively), $\varkappa_1$ and $\varkappa_7$, $\varkappa_2$ and $\varkappa_7$, $\varkappa_2$, and $\varkappa_9$, etc.

Note, however, that if we combine the first and seventh rows, it follows that after that, the first row can no longer be combined with the tenth row, nor can the second row be combined with the seventh. Thus, the compression of the table on the basis of combining a number of rows can be achieved in more than one fashion, and the limiting compression is achieved by examining the various possible entries in the emtpy squares.

Under these conditions the compressed table with the minimal possible number of rows may not be the only such table. Thus, in the example cited above, (Table 12) four different combinations of states are possible which lead to the same minimal number (five) of rows:

I. $\{\varkappa_1, \varkappa_8\}$, $\{\varkappa_2, \varkappa_7, \varkappa_9\}$, $\{\varkappa_3, \varkappa_6\}$, $\{\varkappa_4\}$, $\{\varkappa_5, \varkappa_{10}\}$;

II. $\{\varkappa_1, \varkappa_{10}\}$, $\{\varkappa_2, \varkappa_7, \varkappa_9\}$, $\{\varkappa_3, \varkappa_6\}$, $\{\varkappa_4, \varkappa_8\}$, $\{\varkappa_5\}$;

III. $\{\varkappa_1\}$, $\{\varkappa_2, \varkappa_7, \varkappa_9\}$, $\{\varkappa_3, \varkappa_6\}$, $\{\varkappa_4, \varkappa_8\}$, $\{\varkappa_5, \varkappa_{10}\}$;

IV. $\{\varkappa_1, \varkappa_7\}$, $\{\varkappa_2, \varkappa_9\}$, $\{\varkappa_3, \varkappa_6\}$, $\{\varkappa_4, \varkappa_8\}$, $\{\varkappa_5, \varkappa_{10}\}$.

The result obtained from the combination of rows in Table 12 corresponding to the first variant is shown in Table 13. A dash in a square of the table denotes that for the state $\varkappa_4$ the input ρ_2 cannot appear, and thus the contents of this square do not play a part. The fact that Table 12 cannot be compressed to a number of rows less than five can easily be detected from the fact that among the states $\varkappa_1$, $\varkappa_2$, $\varkappa_3$, $\varkappa_4$, $\varkappa_5$ there is not a single pair of equivalent states.

Assume that we have been able to compress the table down to γ rows. After that we use the table to restore the table for the automaton (we shall call it automaton A^*) and the converter Conv. in exactly the same way as was done at the end of the preceding section (for our example cf. Tables 14 and 15, respectively).

TABLE 13

	ρ_1	ρ_2	ρ_3
$\varkappa_1$	$\varkappa_3$, 6	$\varkappa_4$, 2	$\varkappa_5$, 1
$\varkappa_2$	$\varkappa_3$, 7	$\varkappa_2$, 5	$\varkappa_5$, 1
$\varkappa_3$	$\varkappa_2$, 6	$\varkappa_3$, 3	$\varkappa_2$, 6
$\varkappa_4$	$\varkappa_2$, 6	—	$\varkappa_1$, 3
$\varkappa_5$	$\varkappa_1$, 5	$\varkappa_4$, 2	$\varkappa_1$, 3

TABLE 14

	ρ_1	ρ_2	ρ_3
$\varkappa_1$	$\varkappa_3$	$\varkappa_4$	$\varkappa_5$
$\varkappa_2$	$\varkappa_3$	$\varkappa_2$	$\varkappa_5$
$\varkappa_3$	$\varkappa_2$	$\varkappa_3$	$\varkappa_2$
$\varkappa_4$	$\varkappa_2$	—	$\varkappa_1$
$\varkappa_5$	$\varkappa_1$	$\varkappa_4$	$\varkappa_1$

TABLE 15

	ρ_1	ρ_2	ρ_3
$\varkappa_1$	λ_6	λ_2	λ_1
$\varkappa_2$	λ_7	λ_5	λ_1
$\varkappa_3$	λ_6	λ_3	λ_6
$\varkappa_4$	λ_6	—	λ_3
$\varkappa_5$	λ_5	λ_2	λ_3

A sequential machine constructed according to the scheme shown in Fig. 1 but incorporating automaton A^* instead of the "fast" automaton F realizes the specified automaton A. But now the automaton A^* which contains the constructed machine, no longer operatres with fast cycles (i.e., it does not operate with stable states); rather it operates with the cycles of automaton A (i.e., in cycles determined by changes in the input state). Automaton A^* can therefore be realized from any complete set of elements (for example, it can be constructed from elements which cause a one-cycle delay and instantaneous converters). Automaton A^* has γ states under these conditions.

It is obvious that $\gamma \leq \beta$, since in particular it is possible to fill the empty squares in such a way that the reformulated table coincides with the original (unmodified) flow table. It is also obvious that $\gamma \leq l$, since a realization with l states is always possible as was indicated at the beginning of the section.

On the basis of the above it is possible to determine how to use the additional information concerning the instants at which the input changes introduced into the automaton for the purpose of reducing the number of states; we can also determine how the realization of the automaton from elements operating in the same cyclical manner (for example, using delay elements) is often more advantageous than realizing the automaton from elements operating with "fast" cycles (with stable states).

In Appendix II we demonstrate the relationship between the algorithm cited in this section for compressing the tables and the algorithm for minimization of sequential machines with bounded input sequences [7].

5. THE FURTHER REDUCTION OF THE NUMBER OF STATES IN THE AUTOMATON ON THE BASIS OF INCREASING THE INPUT INFORMATION

We shall return to the scheme which forms automaton A from a "fast" automaton F in accordance with Fig. 1. Assume that δ is the largest number among the numbers α_j (i.e., it is the largest number of different symbols λ that are encountered in any one of the columns in the basic table). Thus, for example, in the case of Table 2, $\delta = 4$. This means that there exist an input state for which δ different symbols λ can appear at the output. But, according to the scheme in Fig. 1, this is possible only when the automaton F has at least δ

different states. Until an output converter is used in the scheme and λ is derived as a logic function of μ and ρ, the number of states of automaton F in principle cannot be made less than δ.

In realizing the automaton according to the scheme shown in Fig. 1 in section 2, we obtained the minimum number of states (which, generally speaking, may be greater than δ). Thus, in the example studied in section 2 we obtained a minimum number of states $\beta = 8$, whereas $\delta = 4$.

We shall assume now (cf. Fig. 2) that both the values ρ (p) and ρ_τ(p) are applied to the input of the automaton (i.e., the values of the input τ seconds before the specified instant are given, where τ is the duration of two "fast" cycles in automaton F).

The instantaneous values of ρ(p) and ρ_τ(p) determine the input state of the automaton. The complete input of this automaton has r^2 states. Therefore, the input alphabet should contain r^2 different symbols. However, we shall make the odd alphabet of $\underline{r}$ symbols suffice; on the other hand, each input state at the specified instant shall be assigned a pair of symbols ρ(p), ρ_τ(p) (in the indicated sequence).

Passing now to the construction of automaton A in accordance with the scheme shown in Fig. 2, we shall illustrate the reasoning above by the example studied in section 3 while keeping the general case in mind.

We shall choose the δ symbols $\mu_1, \mu_2, \ldots, \mu_\delta$. Turning to the first column in the specified basic table for automaton A, we shall assign different symbols μ_j to the different symbols λ_i which appear in that column; we shall assign identical symbols μ_j to identical symbols λ_i appearing in the column. This can always be done, since the number of different symbols λ_i in the column is no greater than δ. Now we shall turn to the second column in the table and shall perform the same operations without associating ourselves in any way with the manner in which the symbols were written in the first column. We shall proceed analogously with all the columns in the table. As a result, each square in the table will contain a symbol μ_j in addition to the symbol λ_i. As an example we cite Table 2 in section 3 with the symbols μ_j written in (Table 16).

From this table we can immediately formulate the table for the output converter in Fig. 2 (Table 17).

TABLE 16

	ρ_1	ρ_2	ρ_3
λ_1	$\lambda_5\mu_1$	$\lambda_2\mu_1$	$\lambda_3\mu_1$
λ_2	$\lambda_6\mu_2$	$\lambda_4\mu_2$	$\lambda_3\mu_1$
λ_3	$\lambda_6\mu_2$	$\lambda_2\mu_1$	$\lambda_6\mu_2$
λ_4	$\lambda_5\mu_1$	$\lambda_3\mu_3$	$\lambda_3\mu_1$
λ_5	$\lambda_7\mu_3$	$\lambda_2\mu_1$	$\lambda_1\mu_3$
λ_6	$\lambda_7\mu_3$	$\lambda_5\mu_4$	$\lambda_1\mu_3$
λ_7	$\lambda_5\mu_1$	$\lambda_3\mu_3$	$\lambda_6\mu_2$

TABLE 17

	ρ_1	ρ_2	ρ_3
μ_1	λ_5	λ_2	λ_3
μ_2	λ_6	λ_4	λ_6
μ_3	λ_7	λ_3	λ_1
μ_4	—	λ_5	—

Here, a portion of the squares may be unfilled; this means that such combinations of ρ and μ are not encountered for the operation of the realized automaton.

The basic table for automaton F contains r^2 columns in accordance with the number of possible combinations of symbols ρ and ρ_τ (in our case the table will contain nine columns; cf. Table 18).

For example, in order to fill the square at the intersection of the row μ_3 and the column ρ_3, ρ_1 in Table 18, we find from Table 17 that the state μ_3 of the input ρ_1 is coded as the symbol λ_7 by the converter. Then from Table 16 we find that for the symbol λ_7 we establish μ_2 due to the effect of the input ρ_3. Thus, we must write μ_2 into the selected square, μ_3, ρ_3, ρ_1. Analogously, we fill all of Table 18 square-by-square. The squares con-

TABLE 18

	ρ_1			ρ_2			ρ_3		
	ρ_1	ρ_2	ρ_3	ρ_1	ρ_2	ρ_3	ρ_1	ρ_2	ρ_3
μ_1	μ_1	μ_2	μ_2	μ_1	μ_1	μ_1	μ_3	μ_1	μ_1
μ_2	μ_2	μ_1	μ_3	μ_4	μ_2	μ_4	μ_3	μ_1	μ_2
μ_3	μ_3	μ_2	μ_1	μ_2	μ_3	μ_1	μ_2	μ_2	μ_3
μ_4	—	μ_3	—	—	μ_4	—	—	μ_3	—

taining dashes denote combinations of states and inputs which cannot be encountered in the automaton for any input sequences.

The scheme shown in Fig. 2 with the converter specified by Table 17 and a "fast" automaton F corresponding to Table 18 realizes the automaton specified by Table 16 if the values of λ and ρ are written in each time during the steady state after ρ has changed.

The number of states in a sequential machine which has been constructed in this manner and realizes the automaton A is therefore equal to δ. Realization using relays can be achieved by any method; for example, a method completely analogous to the $(2s_0 + 1)$-realization developed by Huffman can be used; however, under these conditions we require not s_0 but m binary states, where m is the minimum number satisfying the inequality $2^m \geq \delta$. We shall therefore speak of the $(2m + 1)$-realization of the specified automaton.

Heretofore we assumed that ρ_τ repeats the values ρ with a lag of τ, where τ is the time between two cycles in the "fast" automaton. When realization is achieved with relays, τ is the time required for operation of the relay. Therefore, at first glance it would seem that besides competition between secondary relays we would encounter an additional problem of competition if ρ_τ repeated ρ with a lag that did not exactly coincide with the operate time for the relay. This concept is incorrect. A method of combating competition between relays [for example, Huffman's $(2s_0 + 1$-realization which becomes $(2m + 1)$-realization in our case] provides for the disconnection of all feedback loops during the steady-state period; by eliminating competition between relays the method thus eliminates competition between relays and the device that produces the signals ρ_τ.

At the beginning of this section we demonstrated that it is impossible to realize automaton A according to a scheme such that as shown in Fig. 1, no matter what additional information on the input is available if automaton F has less than δ states. Therefore within the limits of that basic scheme it is possible to produce an automaton which is more economical in terms of the number of internal states (i.e., in terms of the number of secondary relays when the relay method of realization is used). In other words, the information $\rho + \rho_\tau$ on the state of the input is at its upper limit, and a further increase in the information concerning the input is useless.

In conclusion we shall make the following remark. In comparing the three types of realization for the automaton which were described in sections 3, 4, and 5, we see that the condition $\beta \geq \gamma \geq \delta$ is always valid. In particular, in the example under study (Table 2) the required number of states prove to be equal to 8, 5, and 4, respectively.

If the information on the change in ρ which was discussed in section 4 or the information on the value of ρ which is being discussed in this section were itself to be derived from the information on the instantaneous value of ρ using auxiliary finite automata (networks containing secondary relays) then of course it would be necessary to take into account the states of these auxiliary automata as well. But this information can be introduced into the automaton by the same operator or device which is used to introduce the information on the instantaneous

value of ρ; i.e., this information may be just as "external" with respect to the automaton as in ρ. If the same additional information is produced by a special device, then it is essential that this device not be included in the networks of automaton F that are encompassed with feedback loops; this device can be realized by making the input contacts more complex on the basis of completely different engineering means than those used in automaton F. Thus, if the state of any input filament is characterized by the fact that contact a is closed or open, then closure of contact b, which is coupled with contact a by a spring and is equipped with a damper, characterizes the state of that same input with a delay of τ sec. The signal denoting the fact that the input has changed can also be achieved by using purely mechanical means. In those cases, when the input signals are electrical, the information ρ_τ can be organized, for example, using capacitors. In other words, each such device is essentially an input device for the automaton; if, for example, we have relay networks in mind, the input device is no more involved in determining the number of states (or intermediate relays) than are the input or actuating relays in the network.

APPENDIX I

Proof of the theorem. Initially we shall prove the first postulate in the theorem. Assume that a sequential machine M is specified:

$$\varkappa(p) = F[\varkappa(p-1), \rho(p-1)], \tag{8}$$

$$\lambda(p) = \Psi[\varkappa(p), \rho(p)]. \tag{9}$$

We shall formulate a net (cf. [1]) consisting of two finite automata and specified by the equations

$$y(p) = \bar{F}[y(p-1), z(p-1)], \tag{10}$$

$$z(p) = \rho(p-1). \tag{11}$$

and shall specify the output converter

$$\chi(p) = \Psi[y(p), z(p)], \tag{12}$$

where the alphabets $\{y\}$ and $\{\chi\}$ coincide with the alphabets $\{\varkappa\}$ and $\{\lambda\}$, respectively. The alphabet $\{z\}$ differs from $\{\rho\}$ by the addition of one symbol z^0; the function Ψ in (12) coincides with the corresponding function in (9) in terms of all pairs of symbols in $\{\varkappa\}$ and $\{\rho\}$; it is not defined (or is arbitrarily specified) for $z = z^0$. The function $\bar{F}$ in (10) coincides with F in (8) for all combinations of symbols that do not contain z^0; for such combinations we have $\bar{F}(y, z^0) \equiv y$. The equations (10) + (11) describe a finite automaton A whose states are coded by the pair (y, z), and Eq. (12) specifies the output converter of that automaton. Each initial state $\varkappa^0$ of the sequential machine M shall be made to correspond to an initial state in automaton A which is such that

$$y(0) = \varkappa^0, \; z(0) = z^0. \tag{13}$$

For p = 1 it follows from (10) that $y(1) = y(0) = \varkappa^0$. For $p \geq 1$ the symbol z^0 cannot appear, and at these instants $\bar{F}$ can be replaced by F in (10); from this we have

$$y(p+1) = F[y(p), z(p)], \; p \geq 1. \tag{14}$$

Introducing the new variable $Y(p) \equiv y(p+1)$, $(p \geq 0)$ and taking (11) into account, it is possible to write

$$Y(p) = F[Y(p-1), \rho(p-1)], \; p \geq 0, \tag{15}$$

where $Y(0) = y(1) = \varkappa^0$. Equations (8), (15), and the initial conditions $\varkappa(0) = \varkappa^0$, $Y(0) = \varkappa^0$ coincide; in view of this we have

$$Y(p) \equiv \varkappa(p)$$

for any p.

But $y(p) = Y(p-1)$, whence

$$y(p) = \varkappa(p-1), \; p \geq 1. \tag{16}$$

Substituting (11) and (16) into (12) and comparing it with (9), we obtain

$$\chi(p) = \Psi[\varkappa(p-1), \rho(p-1)] = \lambda(p-1), \; p \geq 1,$$

which proves the first postulate in the theorem.

We shall prove the second postulate. Assume that the finite automaton A is specified:

$$\varkappa(p) = F[\varkappa(p-1), \rho(p-1)] \tag{17}$$

with the output converter

$$\chi(p) = \Psi[\varkappa(p)]. \tag{18}$$

We shall make this automaton correspond to the sequential machine M

$$y(p) = F[y(p-1), \rho(p-1)], \tag{19}$$

$$\lambda(p) = \Psi\{F[y(p), \rho(p)]\}, \tag{20}$$

where the alphabets $\{y\}$ and $\{\varkappa\}$ coincide. Each initial state $\varkappa^0$ of automaton A is made to correspond to the initial state $y(0) = \varkappa^0$ of the machine M. Since Eq. (17), (19), and the corresponding initial states coincide, it follows that for all $p \geq 0$ we have

$$y(p) = \varkappa(p). \tag{21}$$

Substituting (21) into (20) and taking (17) and (18) into account, we obtain

$$\lambda(p) = \Psi\{F[\varkappa(p), \rho(p)]\} = \Psi[\varkappa(p+1)] = \chi(p+1)$$

for all $p \geq 0$; this proves the second postulate of the theorem. Thus, the theorem is completely proved.

APPENDIX II

On the minimization of the number of states in a sequential machine when limitations are imposed on the input sequences. In order to prove that the algorithm described in section 4 for compressing tables actually leads to the minimal sequential machine, we shall relate this algorithm to the algorithm developed in [7] for minimization of sequential machines in the case of bounded input sequences.

We shall formulate the diagram of states for the sequential machine which is equivalent to the specified automaton A. This can always be done if the number of circles in the diagram of states is made to equal to the number of rows in the table for the automaton A and if the output converter is assumed identical (i.e., if it is assumed to convert the state with the number j to the output with the same number, irrespective to the input state). Assume, for example, that the table for the specified automaton A is of the form shown in Table 19 (the states are indicated by numbers).

The diagram of states for the equivalent sequential machine is shown in Fig. 5.

TABLE 19

	ρ_1	ρ_2
1	3	3
2	2	3
3	2	1

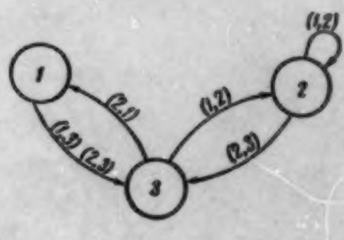

Fig. 5.

We should note one general property of the diagram of states which is formulated in this manner; the second numbers in the pairs which are written above all the arrows pointing toward each circle with the number j coincide with the number of that circle.

The matrix for the connections of the sequential machine whose diagram of states is shown in Fig. 5, is given by

$$
\begin{array}{cc}
 & \begin{array}{ccc} 1 & 2 & 2 \end{array} \\
C = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 0 & (1,3)\vee(2,3) \\ 0 & (1,2) & (2,3) \\ (2,1) & (1,2) & 0 \end{bmatrix}.
\end{array}
$$

The following general property for the connection matrix corresponds to the property of the diagram of states mentioned above: all non-zero elements in any column of the matrix have identical second numbers (which coincide with the number of the column).

The limitations which were examined by Aufenkamp consist of the fact that not all the arrows depart from certain circles on the diagram of states; i.e., for certain states of the machine the appearance of certain input signals is not allowed. In order to make use of the Aufenkamp algorithm, it is necessary to transfer the requirement for forbidding the appearance of two successive identical inputs to the diagram of states. However, this cannot be achieved by simple erasure of certain arrows on the diagram of states, since if two or more arrows with the input numbers a_1, a_2, etc. arrive at the circle with the number $\underline{j}$, then the ability to forbid further movement from the circle $\underline{j}$ depends on which of the arrows the approach was made along. In order to avoid these difficulties we shall construct a sequential machine equivalent to A in a different manner; we shall construct it in precisely such a manner that only arrows with identical first numbers approach each circle. This can be achieved by rearranging the diagram of states for a machine which has already been constructed: each circle which is approached by $\underline{s}$ arrows with different first numbers is replaced by $\underline{s}$ circles to which only arrows with identical first numbers make an approach The second numbers of the approaching arrows remain the same. The same arrows which departed from the original circle depart from these circles. Figure 6 shows the diagram of states of Fig. 5, rearranged according to this rule. It is not difficult to see that the reconstructed sequential machine remains equivalent to the original machine since it remains equivalent to automaton A.

It is necessary to impose the required limitations on the input sequences. In order to do this, we eliminate that arrow from among the arrows departing from each circle on the diagram of states for which the first number coincides with the first number of the arrows approaching the circle. In Fig. 7 we have shown the diagram of states for Fig. 6 if the class of input sequences is bounded – two successive identical inputs are forbidden (we shall call this class R). The matrix for the connections is now given by

$$
\begin{array}{cc}
 & \begin{array}{cccc} 1 & 2 & 3' & 3'' \end{array} \\
C = \begin{array}{c} 1 \\ 2 \\ 3' \\ 3'' \end{array} & \begin{bmatrix} 1 & 0 & (1,3) & 0 \\ 0 & 0 & 0 & (2,3) \\ (2,1) & 0 & 0 & 0 \\ 0 & (1,2) & 0 & 0 \end{bmatrix}.
\end{array}
$$

Each column in the connection matrix for the rearranged sequential machine may contain only identical pairs.

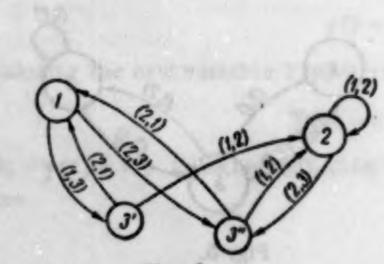

Fig. 6.

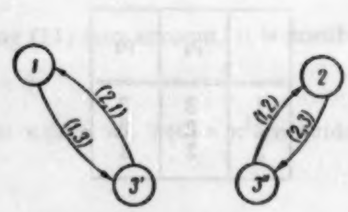

Fig. 7.

The algorithm for recognizing states that are equivalent relative to the class R for matrices of this type is substantially amplified in comparison to the general algorithm described in [7]; it reduces to the following: those rows in the matrix C corresponding to states that are equivalent relative to R form a sub-matrix of the matrix C such that two different rows cannot contain pairs that have identical first numbers and different second

numbers. In our example, the states 1, and 2 or 1 and 3', or 3' and 3", or 2 and 3" are equivalent relative to R, but, states 2 and 3' or 1 and 3" are not equivalent.

Minimization reduces to replacing a group of states that are equivalent relative to R with one state. The basic difficulty resides in formulating a subdivision of all the states that are equivalent relative to R in such a way that the over-all number of groups is minimal; often this minimal number can be determined only by checking all possible variants of the subdivision.

In the simple example cited here it is immediately evident that this minimal number of groups cannot be less than two, since states 2 and 3' are not equivalent and must therefore be in a different group. But, this number is equal to two, since all states can be divided into two groups at least in the following manner: $\{1, 3'\}$ and $\{2, 3''\}$.

The sufficiency of the algorithm proposed in [7] is essentially based on the fact that if the states of the machine are subdivided into equivalent groups, then there exists the symmetrical subdivision of the connection matrix required in [7]. Although the proof of this fact is not cited in [7], it is nevertheless true that the existence of such a subdivision can easily be established for a connection matrix of the type which we are studying.

The algorithm described in section 4 is a simple algorithm and a more convenient modification of the Aufenkamp algorithm.

SUMMARY

There are considered three methods of realization, which are economic from the viewpoint of number of states, of a finite automaton set by an equation (or table) and operating in a time pace regime caused by change of the input states of the automaton. The methods are distinguished by an amount of information, which defines the automaton input. The first method (Huffman method) intends minimum information, i.e., information on input state at the moment only; the second method supposes the introduction into the automaton of additional information on moments of changing input state; the third method intends introduction into the automaton of information on input state at the moment and at some preceding moment of time.

It is shown that one can get the most economic automaton using the third method.

LITERATURE CITED

1. M. A. Aizerman, L. A. Gusev, L. I. Rozonoer, I. M. Smirnova, and A. A. Tal', "Finite automata, I", Avtomatika i Telemekhanika, 21, 2 (1960).*
2. M. A. Aizerman, L. A. Gusev, L. I. Rozonoer, I. M. Smirnova, and A. A. Tal', "Finite automata, II", Avtomatika i Telemekhanika, 21, 3 (1960).*
3. E. F. Moore, "Speculative experiments with sequential machines," Symposium on Automata, edited by A. A. Liapunov [Russian translation] (Foreign Literature Press, Moscow, 1956).
4. A. Sh. Blokh, "On problems that are solved on sequential machines," Symposium on Cybernetics Problems, No. 3, Edited by A. A. Liapunov [in Russian] (Physics-Mathematics State Press, Moscow, 1960).
5. D. D. Huffman, The Synthesis of Sequential Switching Circuits, Journ. Francl. Inst. vol. 257, No. 3, 4, 1954.
6. D. D. Aufenkamp and F. E. Hon, Analysis of Sequential machines, in: Mathematics (Periodic Symposium of Translations of Foreign Papers 3: 3, 1959).
7. D. D. Aufenkamp, Analysis of Sequential machines, in: "Mathematics" (Periodic Symposium of Translations of Foreign Papers, 3: 6, 1959).

*See English translation.

TRANSISTORIZED RELAY NETWORKS

A. P. Suponitskii and V. D. Urin

(Moscow)

Translated from Avtomatika i Telemekhanika, Vol. 21, No. 12,
pp. 1595-1600, December, 1960
Original article submitted May 30, 1960

The paper analyzes the operation of transistorized relay networks with diode input converters. The special features of thermal stabilization in such networks are analyzed. The basic conclusions are verified experimentally.

The use of transistors in relay networks is an effective means of improving the sensitivity and lowering the power requirements in such networks. Transistorized relay networks can be operated from the control-signal source with a low power consumption. It is easy to change the settings in such networks. The networks are reliable for prolonged operation both in the "energized" and "de-energized" states. In many cases the operating conditions for such networks involve a narrow temperature range (20° ± 10°).

Most often the common-emitter transistor connection is used in relay networks.

Many relay networks compare a monitored voltage U_1 that is produced, by a detecting network, for example, with a standard voltage U_2. Each of these voltages can in the general case vary from zero to a maximum value. The monitored voltages are in many cases appreciably greater than the allowable voltage rating of the transistors. Therefore, it proves necessary to make use of such devices as, for example, a voltage divider network (Fig. 1).

These networks can be designed in two ways. In the first method the voltages are first compared and then their difference is divided down to a practical magnitude. In the other method, the monitored voltage is first divided in a definite ratio and is then compared with the set voltage. For networks which do not contain nonlinear elements both methods are equivalent. However, the second method permits simple introduction of rectifiers into the network, and these rectifiers can provide the property of diode limiting (Fig. 2). The presence of rectifiers makes three operating modes possible for the network:

1) both of the rectifiers B_1 and B_2 (as well as the transistor) are conducting;

2) the rectifier B_1 conducts, the rectifier B_2 and the transistor are cut off;

3) the rectifier B_1 is cut off, the rectifier B_2 and the transistor are conducting.

The first mode is the "working" mode. Under these conditions the operation of the network shown in Fig. 2 is fully equivalent to the operation of the network shown in Fig. 1. However, the presence of two other modes of operation for the networks shown in Fig. 2 makes it possible for us to approach the selection of parameters in a different way. It is not difficult to see that in the second mode the base current is $I_2' = 0$, and in the third mode it is $I_2' = U_2/(R_b + R_{ol})$. The network is able to protect the transistor from excessive voltages across the emitter-base junction and from overload. In the second mode of operation, this is achieved by means of the backward resistance of the rectifier B_2 (it is desirable to choose the rectifier in such a way that its backward resistance appreciably exceeds the backward resistance of the emitter-base junction). The protection of the transistor from overload in the third mode of operation is achieved by making the value of R_{ol} sufficiently large to limit the current through the transistor for the maximum value of the voltage U_2.

Since the network in Fig. 1 does not have such protective properties, the only method of limiting the current through the transistor involves lowering the sensitivity.

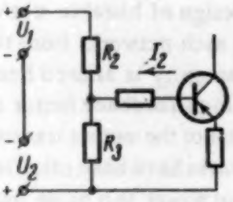

Fig. 1.

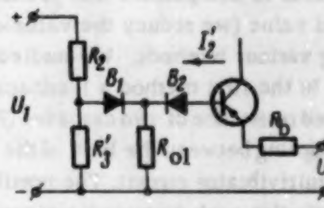

Fig. 2.

In fact, assume the parameters of the networks are chosen in such a way that the values of R_2 and R_b are identical in both cases, and $R_3 = R_3'R_{ol}/(R_3' + R_{ol})$. Then the input resistance of the networks from the U_1 side will also be the same. For the network in Fig. 1 the base current is $I_2 = (U_2 - U_1)R_3/(R_2R_3 + R_2R_b + R_3R_b)$, and for $U_2 = U_{2max}$, $U_1 = 0$, it reaches its maximum value

$$I_{2\,max} = \frac{U_{2\,max}}{R_2 + R_b + \dfrac{R_2R_b}{R_3}}.$$

For the network in Fig. 2 operating in the third mode,

$$I'_{2\,max} = \frac{U'_{2\,max}}{R_{ol} + R_b},$$

where $U'_{2\,max} = U_{2\,max}\dfrac{R_3}{R_3 + R_2}$.

Therefore,

$$I'_{2\,max} = \frac{U_{2\,max}R_3}{(R_2 + R_3)(R_{ol} + R_b)} = I_{2\,max}\frac{R_2R_b + R_3R_b + R_2R_3}{R_2R_b + R_3R_b + R_2R_{ol} + R_3R_{ol}}.$$

The value of R_b must be less than R_2, R_3 (since an increase in R_2, R_3, not only lowers the sensitivity but also the power consumption, and an increase in R_b basically lowers the sensitivity without having much effect on the power consumption).

It is also obvious that $R_{ol} > R_3$. Therefore, it is always true that $I'_{2\,max} < I_{2\,max}$. A lowering of $I_{2\,max}$ (i.e., an increase in $R_2 + R_b + R_2R_b/R_3$) is possible only when the sensitivity is lowered, whereas the value of R_{ol} can be selected over wide limits for the condition that there is a corresponding variation in R_3' without worsening the sensitivity; this is true because the current I_2' depends only on the equivalent resistance $R_3 = R_3'R_{ol}/R_3' + R_{ol}$. Thus, the protective properties of the network in Fig. 2 permit a wider choice of parameters.

In performing experimental verifications (cf. below) we used the network in Fig. 2. Eliminating the rectifiers B_1 and B_2 from the network is practically equivalent to converting to the network shown in Fig. 1; if all the other parameters were to remain constant, this would cause an increase in the maximum current through the base of the input transistor (for $U_{2\,max}$ and $U_1 = 0$) by a factor of approximately two (compared to the actual results); this is clearly undesirable. The use of a diode limiting network without voltage division at the input (for, $R_2 = 0$, $R_C' = \infty$) is not always possible in the problem under study. The input resistance of the network shown in Fig. 2 from the U_1 side is equal to $R_{1\,x} = R_2 + R_b$ (in the first mode of operation). It is assumed that the internal resistance of the source U_2 is included in the value of R_b.

Thus, a reduction in the input resistance when R_2 is removed could be compensated only by increasing R_b on the basis of a corresponding increase in the resistance of the transistor control circuit (the emitter-base resistance) in the first mode of operation for $R_2 = 0$, $R_C = R_b + R_{i1}$, where R_{i1} is the internal resistance of the source U_1. The latter result, as we shall indicate below, is undesirable.

The relationships for the network in Fig. 2 in the second and third modes of operation are derived for ideal rectifiers. The presence of a backward resistance in the rectifiers which is commensurate with the resistors R_3', R_{ol}, R_2 violates these relationships. However, the selection of the parameters is substantially affected only by the leakage of a portion of the current through the backward resistance of the rectifier B_1 in the third mode of operation. In order to compensate this phenomenon the value of R_{ol} must be increased appreciably compared to the computed value (we reduce the value of R_3'' correspondingly). The design of bistable static relay networks is possible using various methods. We studied two methods for constructing such networks from two-stage transistor amplifiers. In the first method a feedback factor equal to or greater than unity is assured because of the aggregate emitted resistance of two cascades (Fig. 3). In the second method a high feedback factor is assured because of the mutual coupling between the base of the input transistor and the collector of the output transistor (Fig. 4); this is analogous to a multivibrator circuit. The possibilities of the first of these networks have been clarified in the literature [1] where, in particular, such a network was used to obtain the maximum output power in a pulse mode of operation.

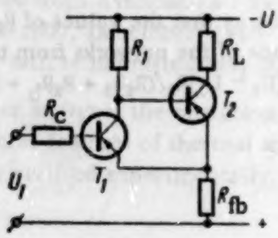

Fig. 3.

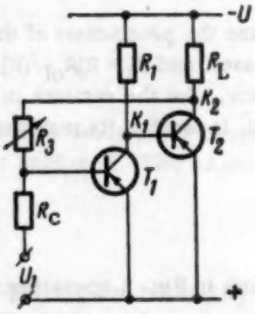

Fig. 4.

With respect to the problems under study, we found that the possibilities of using this network were limited. In accordance with [1] the relay mode of operation for the network is achieved for the condition $R_{fb} - \frac{R_c}{\beta_1\beta_2} > 0$ where R_{fb} is the feedback resistance, R_c is the normalized resistance of the control circuit, β_1 and β_2 are the values of current gain for the transistors T_1 and T_2. The supply voltage is

$$U = I_L(R_L + R_{fb}).$$

The relationships cited above show that an increase in R_c leads to an increase in R_{fb}; this, in turn, leads to an increase in the supply voltage U (for specified values of load current I_L and load resistance R_L) and also to a lowering of the efficiency of the relay networks. For a specified U, an increase in R_{fb} leads to a lowering of the output transistor gain and thus lowers the sensitivity of the over-all network.

Since the allowed voltage across the collector of the transistors is limited, the value of R_c is also limited. The limiting of R_c leads to the limiting of R_{in} since

$$R_{in} = R_c\frac{R_3 + R_b}{R_b + \frac{R_2R_3}{R_2 + R_3}} = R_c\left(1 + \frac{R_2^2}{R_b(R_2 + R_3) + R_2R_3}\right).$$

It should be noted that this limitation has an effect in the case where the input-signal source has a high internal resistance. The network shown in Fig. 4 is free from this limitation. The feedback factor for the network in Fig. 4 can be found as follows.

Assume the current through the base of the input transistor has varied by the amount $+\Delta i_{b1}$. This leads to a variation in the collector current of the input transistor by the amount $+\beta_1\Delta i_{b1}$. It can be assumed that an insignificant variation of the current is accompanied by a considerably smaller variation of the potential at the point K_1 (a variation of the potential at this point corresponding to the entire working range of the current over the linear segment is equal to tenths of a volt). Therefore, it can be assumed conditionally that the potential at the point K_1 does not change. This means that the current through the base of the output transistor must vary by the amount $-\beta_1\Delta i_{b1}$, while its collector current varies by the amount $\beta_1\beta_2\Delta i_{b1}$.

If we disconnect the feedback, then the potential at the point K_2 would change by the amount $\beta_1\beta_2\Delta i_{b1}R_L$. Correspondingly, using the theorem for an equivalent generator, the current in the feedback loop would increase by the amount $+\Delta i'_{b1} = \Delta i_{b1}\beta_1\beta_2R_L/(R_3 + R_L)$ in that case. On the basis we find the feedback factor:

$$K_{fb} = \frac{\Delta i'_{b1}}{\Delta i_{b1}} = \frac{\beta_1\beta_2R_L}{R_3 + R_L};$$

for $R_L \ll R_3$

$$K_{fb} \approx \frac{\beta_1\beta_2R_L}{R_3}.$$

From the expression for K_{fb} it follows that the adjustment of both K_{fb} and the gain for the entire network can be achieved by varying the magnitude of R_3. The value of R_3 is bounded from below on the basis of the concepts governing the allowable current through the load when the output transistor is cut off. The condition governing a relay mode of operation $K_{fb} \geq 1$ is not related to any limitations on R_c and R_{in}.

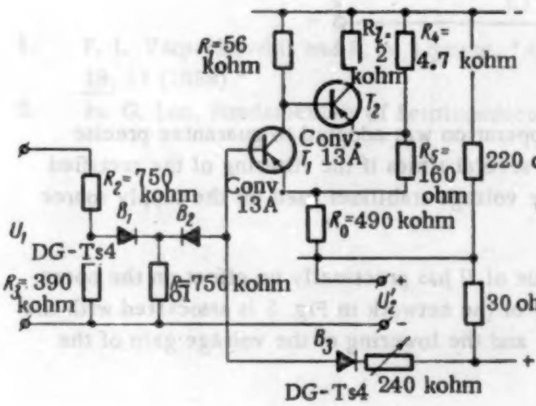

Fig. 5.

Below we cite the data for tests performed on relay networks which have the properties indicated at the beginning of the paper. The network in Fig. 5 is a development of the network shown in Fig. 3; the network in Fig. 6 is a development of the network shown in Fig. 4. In developing the relay network shown in Fig. 5 we made a number of modifications from the recommendations made in [1].

1. The supply voltage for the transistors was lowered appreciably, since operation at the maximum allowable voltage may lower the stability of the settings.

2. In order to achieve guaranteed cutoff of the output transistor we provided for automatic bias by means of a voltage divider (instead of a separate source of bias voltage between the collector of the input transistor and the base of the output transistor as proposed in [1]); this made the network simpler and more reliable.

3. The temperature stabilization of the operating point was achieved using a counter-connected diode [2]. The backward plate resistance of such a diode decreases with an increase in temperature; this increases the current in the diode circuit and compensates the increase in the initial collector current through the input transistor which accompanies an increase in temperature. Laboratory tests showed that the results were stable. This method of thermal stabilization for a variation of 20°C in the ambient temperature practically assured the maintenance of the values of the "operate" and "release" voltages for the relays (Fig. 7, curves a). Without thermal stabilization a temperature increase of 20°C led to a shift of more than 4% of the working range for all of the relay characteristics in the direction of larger values of the control signal (Fig. 7, curves b).

A shortcoming of this method of thermal stabilization is the necessity of using two supply sources or of using a voltage divider (i.e., it is necessary to waste a portion of the power supplied from the sources). The temperature stabilization in the network shown in Fig. 6 was achieved by including a counter-connected diode with a shunt resistance in the divider circuit. This methdod is applicable both for the divider network shown in Fig. 1 and for the diode-limiting network shown in Fig. 2. In our example, a temperature variation of 20°C had practically no effect on the relay characteristic (cf. Fig. 7, curves c). Here, just as in the network shown in Fig. 5, we used automatic bias for the output transistors.

A shortcoming of this method of thermal stabilization is the fact that it can be used only with a high-resistance divider. For the examples cited above the power drawn from the source of the detected signal is twice as

small in the network shown in Fig. 6 as it is in the network shown in Fig. 5. For an upper boundary of 100 v for the voltage of the measured signal, the current drawn by the network in Fig. 6 was 0.06 to 0.07 ma; the current drawn by the network in Fig. 5 was 0.13 to 0.14 ma. The loop between the operate and release voltages U_1 was equal to approximately 2 to 3 v; i.e., it corresponded to a current variation of 2 μa for the network in Fig. 6, and 3 μa for the network in Fig. 5.

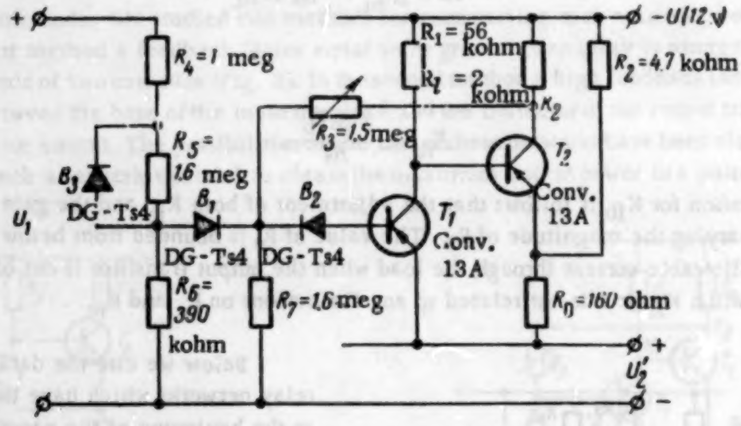

Fig. 6.

The indicated value for the loop corresponding to voltage operation was adopted to guarantee precise operation of the relays. The loop can be narrowed by a factor of several times if the filtering of the rectified voltages U, U_1, and U_2 is improved, and if the effectiveness of the voltage stabilizer used for the supply source is increased.

It should be noted that a variation of ± 10% in the magnitude of U has practically no effect on the operation of the network shown in Fig. 6. The lower current sensitivity of the network in Fig. 5 is associated with the loss of a portion of the supply voltage across the feedback resistor and the lowering of the voltage gain of the output transistor.

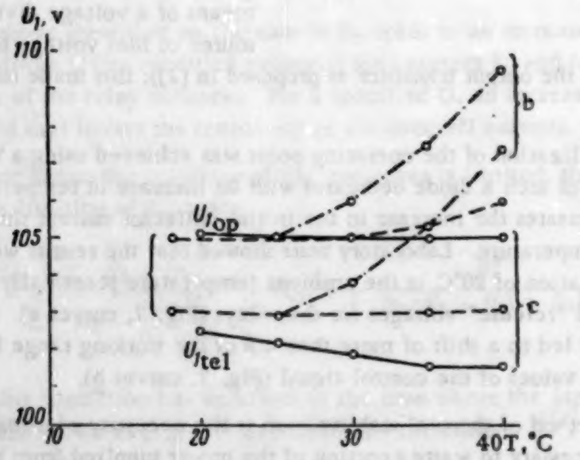

Fig. 7.

CONCLUSIONS

1. The use of a diode-limiting network which protects the transistor from overloads facilitates the selection of the parameters for the input section of a relay network and makes it possible to achieve a higher sensitivity.

2. The design of transistorized relay networks having the properties cited at the beginning of the paper is possible when various methods are used for introducing feedback. The introduction of feedback from the collector of the output transistor to the base of the input transistor offers greater possibilites than the use of a common emitter resistance, since it permits a better utilization of the transistor gains. This conclusion is of a particular nature and is applicable for the statement of the problem formulated in this particular paper.

3. The thermal stabilization of transistorized relays on the basis of using diodes as "thermistors" is fully adequate over a narrow range of temperature variation.

SUMMARY

Transistor relay circuits are considered. Operation of the said circuits with a diode inverter at the input is analyzed. The peculiarities of thermal stabilization of these circuits are studied. Main results are confirmed by an experiment.

LITERATURE CITED

1. F. L. Varpakhovskii and R. A. Lipman, "A contactless transistorized relay," Avtomatika i Telemekhanika 19, 11 (1958).*
2. M. G. Lou, Fundamentals of Semiconductor Electronics [in Russian] (Soviet Radio Press, 1958).

*See English translation.

A THREE-CHANNEL MULTIPLIER WITH FREQUENCY DIVISION OF THE SIGNALS

A. A. Maslov
(Moscow)
Translated from Avtomatika i Telemekhanika, Vol. 21, No. 12,
pp. 1601-1612, December, 1960
Original article submitted May 29, 1960

The paper makes a survey of multipliers of the combination type with amplitude of the channels ("coarse-fine" systems).

A new design is proposed for a multiplier with frequency division of the input signals; the properties of the device are studied, and data is cited for an experimental verification of one variant of the network based on reversive stepfinders.

An analysis of various principles for designing multipliers [1] showed that based on known principles it is not possible to design a multiplier which for a comparatively high accuracy (0.1 to 0.01%) also provides adequate dynamic properties. Certain attempts at expanding the pass band of multipliers while retaining high accuracy by making a transistion to coarse-fine systems have been undertaken by a series of authors. Thus, for example, A. A. Fel'dbaum and L. N. Fitsner developed [2] the principle of amplitude division of the channels in a multiplier (coarse-fine systems).

Several variants of multiplier networks of this type were studied in the paper by L. N. Fitsner [3]. Then G. M Petrov introduced a series of improvements into networks with amplitude division of the channels. He proposed a method for improving the dynamic properties of coarse-fine systems by introducing nonlinear transfer coefficients into one of the channels.

In all proposed networks of the combined type it was possible to achieve a static accuracy of up to 0.05% and even higher, but the pass band remained considerably lower than the frequency range of the "fine" section of the system. Thus, the basic contradiction between accuracy and speed of response in systems with amplitude division of the channels was not eliminated.

In the Institute of Automation and Remote Control of the Academy of Sciences, USSR we have for several years been developing the idea of designing computers and their elements on the basis of using parallel computing channels with frequency division of the signals.

This paper is a part of an over-all research program being conducted under the supervision of B. Ya. Kogan in this field as it applies to multiplier-divider units. This paper demonstrates that the principle of frequency division of channels makes it possible to design a multiplier with engineering characteristics that are no worse than those of linear decision elements. In order to bring out the engineering advantages of the principle of frequency division over other principles, we shall make a survey of networks with amplitude division of the channels. Then we study the engineering possibilities offered by the new block diagram for multiplier units and cite from an experimental verification of a particular variant using stepfinders.

1. COARSE-FINE MULTIPLIERS

The over-all block diagram of this type of device is shown in Fig. 1. The multiplier U_x is converted to a transfer coefficient for the elements α_1 and α_2 by the converter Conv.; this is done in such a way that $\alpha_1 = \alpha_2 =$

$= U_x^*/U_k$. The conversion of U_x into α_1 and α_2 is usually achieved using servosystems with step variations of the transfer coefficients; therefore the voltage U_x^* is equal to U_x with an accuracy of up to the error caused by the discrete nature of the converter. The error voltage $U_x - U_x^*$ is amplified by a factor K and then multiplied by U_y/U_k in the multiplying device M. The result is divided by K and is entered as one of the terms $U_{z1} = (U_x - U_x^*)/U_k$ in the adder Σ_2.

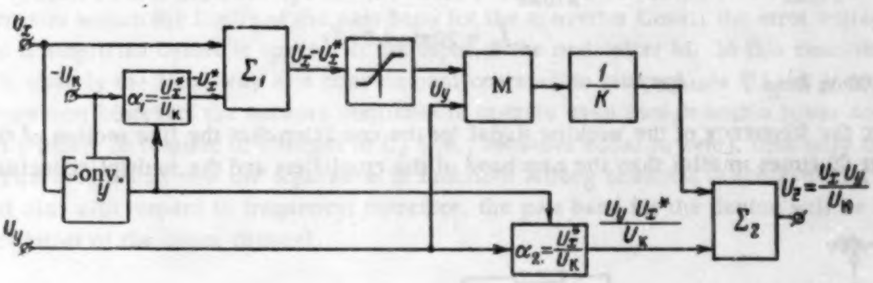

Fig. 1.

In the lower portion of the network the multiplier U_y is multiplied by $\alpha_2 = \alpha_1$; then the resulting voltage $U_{z2} = \frac{U_x^* U_y}{U_k}$ is added to U_{z1}. The total voltage is $U_z = U_{z1} + U_{z2} = \frac{U_x U_y}{U_k}$.

The static accuracy with which the production U_z can be obtained is determined by the sum of the errors of each of the terms; however, when electromechanical converters are used this error will depend basically on the error with which U_{z1} is obtained (this error is K times smaller than the error of the multiplier M).

The pass band of coarse-fine networks is usually assumed to be equal to the pass band of the coarse portion of the network, since the multiplier M is chosen with an adequate speed of response. In fact the pass band for the upper section of the network in Fig. 1 is appreciably lower than that of the multiplier M; therefore, our assertion is valid only in two cases: a) when the number of steps in the discrete section of the network is sufficiently great and turning off the upper channel leads to an insignificant increase in the dynamic error, or b) when the speed of response of the lower section of the network is several orders of magnitude smaller than the pass band of the multiplier M.

In order to clarify the effect of each of the channels on the dynamic properties of such devices, we shall study a specific example.

As our example, we shall choose the Fitsner [3] network which has the greater speed of response. The coarse section of this network (Fig. 2a) contains ten symmetrical triggers Tr1 to Tr10 with operating voltages of 0, 10, 20, . . . , 90 v. The plate circuits of the trigger tubes contain relays whose contacts change the admittancesof the input circuits of the amplifiers A_1 and A_2. The relay of trigger Tr1 which operates for $U_x = 0$ switches the polarity of the voltages U_x and U_y which are applied to the network in four quadrants. The multiplier in the fine section of the network consists of a multiplying block based on thyrite resistors [4].

The number of discrete values of the controlled admittances in the network is chosen equal to ten; therefore, the accuracy of the network can be made ten times higher than the accuracy of the thyrite multiplier (i.e., 0.05 to 0.1%).

The small number of steps made it possible to both avoid using a binary counter or a stepfinder and to make the transition to a network with parallel switching of the triggers. Because of this, the speed of response for the coarse section of the network is limited only by the "operate" time of a single relay.

The dynamic error of the fine section of the network depends on the nature of the signal $K(U_x - U_x^*)$ at the output of the amplifiers A_1 and A_2. If, for example, we apply the signal $U_x = U_{x\,max} \sin \omega t$, to the input of the multiplier, then the voltage U_1 at the output of the amplifier A_1 will vary as shown in the graph in Fig. 2b. The frequency and shape of the converted voltage U_1 are variable and depend on the rate of change of the signal U_x. The relationship between the frequency f_1 of the first harmonic of U_1 and the time rate of change dU_x/dt can be expressed using the relationship

$$\frac{dU_x}{dt}\,\frac{1}{f_1} = \Delta U_T = 10 \text{ v.}$$

(Here ΔU_T is the interval between the "operate" voltages for the triggers).

For $U_x = U_{x\,max} \sin \omega t$ we obtain $U_{x\,max}\, 2\pi f \cos 2\pi f t = 10 f_1$, and from this we have

$$f_1 = 20\pi f \cos 2\pi f t, \tag{1}$$

for $U_{x\,max} = 100$ v; here $f = \omega/2\pi$.

Therefore the frequency of the working signal for the condition that the fine section of the system operates must be at least 60 times smaller than the pass band of the amplifiers and the multiplier included in this section.

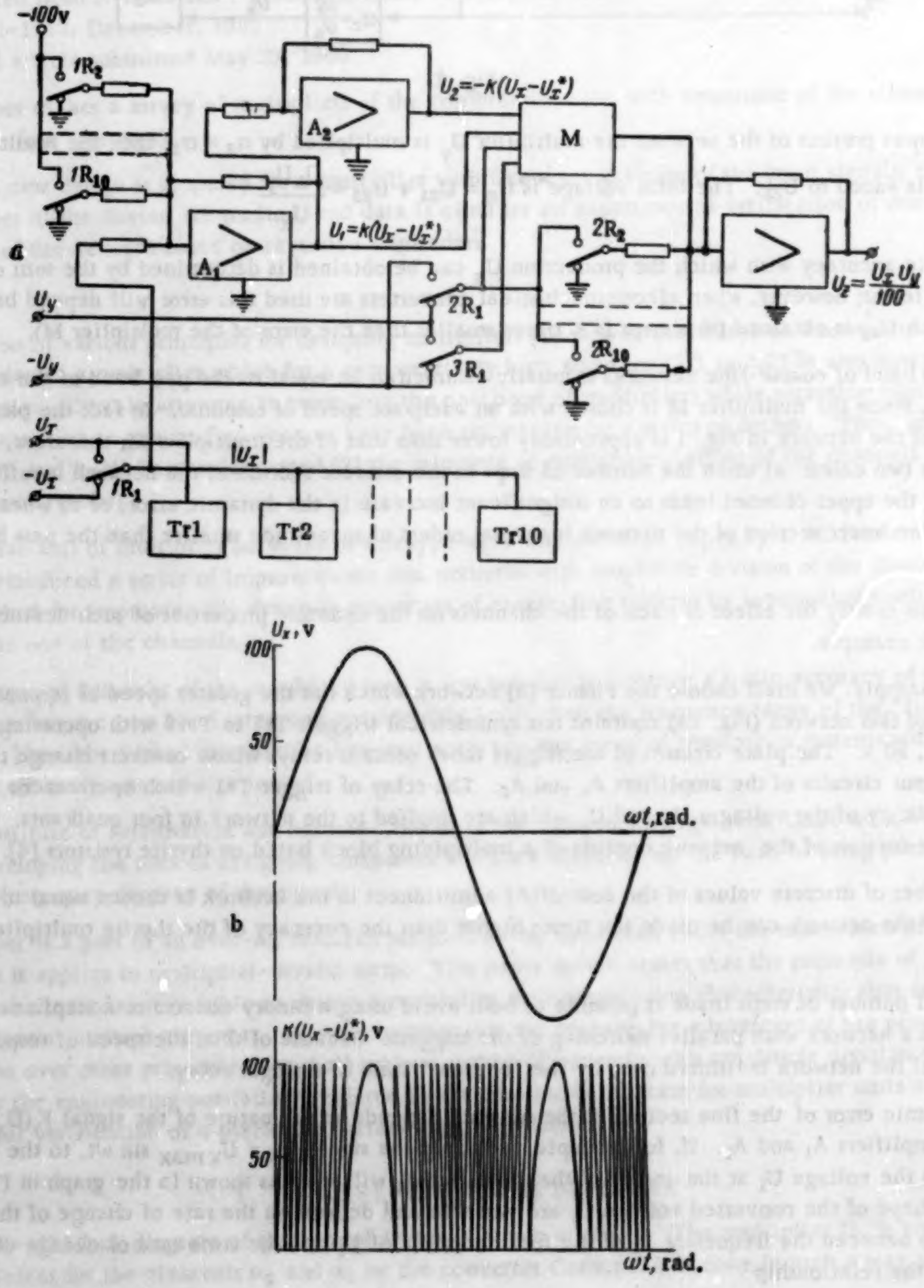

Fig. 2.

The complex shape of the converted signal U_1 (this signal involves the presence of a large number of high-frequency harmonics with a slowly decaying amplitude) begins to have a negative effect even at very low frequencies. Usually this is manifested in the appearance of brief pulses at the output of the device at the instants that the relays commutate.

The block diagram for a multiplier with a nonlinear transfer function (this was proposed by G. M. Petrov; cf. Fig. 3) differs from the preceding network only in the arrangement of the upper channel. While the rate of change of U_x remains within the limits of the pass band for the converter Conv., the error voltage $U_x - U_x^*$ remains small and is amplified before it appears at the input of the multiplier M. In this case, the network as a whole behaves in exactly the same way as a conventional coarse-fine systems. As dU_x/dt increases, the difference $U_x - U_x^*$ also increases; however, the network continues to operate even though with a lower accuracy. When the converter ceases entirely to respond to changes in U_x (i.e., becomes equal to zero), then only the upper channel will function. This means that now the separation of functions among channels is achieved not only with respect to amplitude but also with respect to frequency; therefore, the pass band for the device will be determined by the dynamic properties of the upper channel.

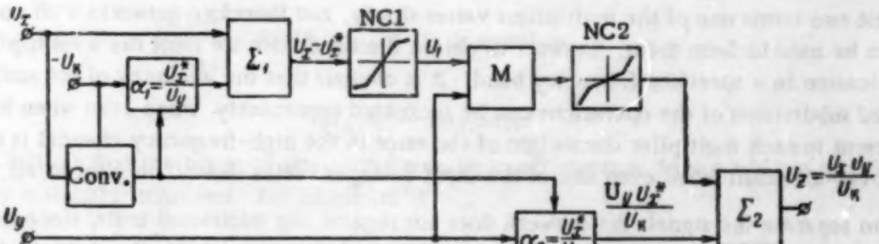

Fig. 3.

The accuracy of the multiplication after the lower channel has been turned off will be somewhat less than the accuracy of the multiplier M since the magnitude of the voltage at the output of the nonlinear converter NC1 (Fig. 3) cannot exceed the linear range of the decision amplifier; thus the transfer coefficient for NC1 corresponding to large values of $U_x - U_x^*$ must be made less than unity. Since the slope of the corresponding segment of the characteristic for NC2 must be made greater than unity, it follows that the error in the multiplication device M will be amplified.

The pass bands of the network can be assumed approximately one order of magnitude greater than the pass band of the multiplier device M. The growth of the dynamic error for the network occurs due to the appreciable expansion of the spectra for the intermediate voltages by the function generators NC1 and NC2. If, for example, $U_x = U_{x\,max} \sin \omega t$, then the signal U_1 at the input of NC1 can be represented by the sum of the same sinusoidal signal with a smaller amplitude and a signal in the form of a trapezoid (Fig. 4). The resulting sum is approximately given by the following Fourier series:

$$U_1(t) = \left(\frac{4aU_{x\,\max}}{\pi} + bU_{x\,\max}\right)\sin \omega t + \frac{4aU_{x\,\max}}{\pi}\left(\frac{\sin 3\omega t}{3} + \frac{\sin 5\omega t}{5} + \ldots\right),$$

where a, b are proportionality coefficients.

As the frequency $f = \omega/2\pi$ increases, the higher harmonics will be suppressed more and more by the multiplying device M, and the signal at its output will approximate the shape of the sinusoidal signal. After the resulting product $U_1(t)\,U_y$ has been multiplied by the transfer function of NC2, the shape of the voltage U_z will be appreciably distorted (cf. Fig. 4b). Therefore, the transition to frequency division of the channels using nonlinear transfer functions does not permit full utilization of the dynamic properties of the multiplying device in the high-frequency channel. Another shortcoming of the network is the low accuracy in multiplying an ac voltage U_x by zero and by small values of U_y.

2. A THREE-CHANNEL MULTIPLIER WITH FREQUENCY DIVISION OF THE CHANNELS

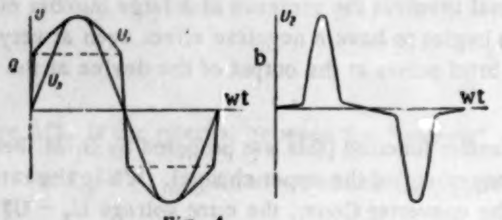

Fig. 4.

In developing the network we used a very valuable quality of many multiplying devices with a controlled transfer coefficient – they are almost completely inertialess with respect to one of the channels while still assuring high accuracy.

In the block diagram of the unit under study (cf. Fig. 5) this quality is realized by subdividing each multiplier into two components: a low-frequency and a high-frequency component (i.e., $U_x = \overline{U}_x + \tilde{U}_x$ and $U_y = \overline{U}_y + \tilde{U}_y$). Under these conditions the products can be written as the sum of three terms:

$$U_z = U_{z1} + U_{z2} + U_{z3} = \frac{\overline{U}_x \tilde{U}_y + \tilde{U}_x \overline{U}_y + \tilde{U}_x \tilde{U}_y}{U_K}. \tag{2}$$

In the first two terms one of the multipliers varies slowly, and therefore networks with controlled transfer coefficients can be used to form them. In order to obtain the third term we must use a multiplying device which assures multiplication in a specified frequency band. It is obvious that the accuracy of the multiplying device for the indicated subdivision of the operations can be increased appreciably since even when high-frequency components are present in each multiplier the weight of the error in the high-frequency channel is reduced; during periods in the over-all result when even one of the input voltages varies slowly, no error at all will be present.

In order to separate the signals the network does not require any additional units, since the converters Conv. 1 and Conv. 2 respond only to the low-frequency components of the over-all signals while the high-frequency components represent the errors of the servosystems. Each of the three channels shown in the block diagram in Fig. 5 can be detached from the system and used as an independent multiplying device. When one or two channels are detached from the network, the network still remains operable, but its engineering characteristics deteriorate.

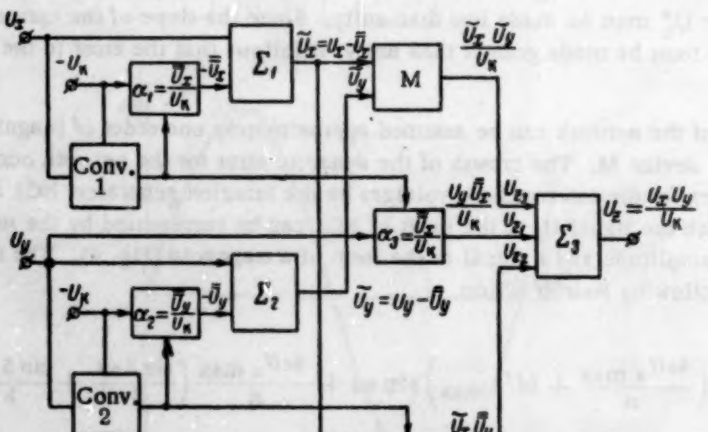

Fig. 5.

We shall study how the accuracy and speed of response of the network are affected when various numbers of channels are used. The error due to the fact that the transfer coefficients α_1, α_2 are not identical to the transfer coefficients α_3, α_4 shall be neglected for the time being, since this error is extremely insignificant in electromechanical multiplying devices. The error in setting the magnitude of the transfer coefficients for the servosystem (this error, for example, is caused by an error in the quantization, by the dynamic error, or by other factors)

is assumed to equal $\delta\alpha_s$. The error of the multiplying device M shall be assumed equal to zero when one of the multipliers $\tilde{U}_x$ or $\tilde{U}_y$ is absent;* the error is equal to δU_M when the voltage at the output of the multiplier M is greater than $\delta U_M U_{z\,max}$.

When only one of the channels in the network is turned on, the network properties are quite obvious. If the network contains two electromechanical multiplying devices, then by assuming that the voltage at the output of each of the channels is equal to the sum of the ideal value of the product and the error, we can write

$$U_{z1}=\frac{U_xU_y}{U_k}+U_y\delta\alpha_{3s},\ \tilde{U}_x=U_x-U_k\alpha_1.$$

Assuming $\alpha_1 = \alpha_{1i} + \delta\alpha_{1s}$ and $U_x - U_k\alpha_{1i} = 0$, where α_{1i} is the ideal value of α_1, we obtain $\tilde{U}_x = -U_k\delta\alpha_{1s}$.

Representing the error in the transfer coefficients α_1 through α_4 analogously, we obtain:

$$U_{z2}=\tilde{U}_x\alpha_4=-U_y\delta\alpha_{1c}-U_k\delta\alpha_{1C}\delta\alpha_{4S},$$

$$U_z=U_{z1}+U_{z2}=\frac{U_xU_y}{U}-U_k\delta\alpha_{3C}\delta\alpha_{4S}.$$

The relative error $\delta U_z = \dfrac{\Delta U_z}{U_{z\,max}}$ for $U_{z\,max} = U_k$ is equal to

$$\delta U_z=-\delta\alpha_{3s}\delta\alpha_{4s}. \tag{3}$$

From this it follows that the errors in setting the transfer coefficients α do not add but multiply; therefore, the static accuracy is sharply improved. For example, if

$$\delta\alpha_{3s}=\delta\alpha_{4s}=0.5\%,\ \text{then}$$
$$\delta U_z=0.0025\%.$$

If one of the multipliers varies at such a rate that the servosystem ceases to operate (i.e., $\delta\alpha_s = 1$), then the multiplication of a slowly varying quantity by a variable quantity will be achieved with the accuracy of the operating channel.

Thus, a two-channel system incorporating two electromechanical multiplying devices assures inertialess multiplication with respect to each of the multipliers (when the other varies slowly) and permits an appreciable increase in the static accuracy.

The graphs for the variation of the error as a function of frequency for a sinusoidal signal in the case where an alternating voltage is multiplied by a slowly varying voltage and for the case where an alternating voltage is squared are shown in Figs. 6a and 6b.

In practical units the accuracy within the limits of the pass band f_{lim1} of the servosystem proves to be less than that shown in the graphs, since in formula (3) we have not taken into account the error due to the fact that the coefficients α are not identical. If the error due to the fact that the element α_3 is not identical to the element α_1 is equal to $\delta\alpha_{3n}$, and the error due to nonidentity between the elements α_4 and α_2 is equal to $\delta\alpha_{1n}$, then the error can be determined from the expressions

$$U_{z1}=\frac{U_xU_y}{U_k}+U_y(\delta\alpha_{3s}+\delta\alpha_{3n}),$$

$$\tilde{U}_x=U_x-U_k\alpha_1=-U_k\delta\alpha_{1s},$$

$$U_{z2}=\tilde{U}_x\alpha_4=-U_y\delta\alpha_{1s}-U_k\delta\alpha_{1s}(\delta\alpha_{4s}+\delta\alpha_{4n}),$$

$$U_{z1}+U_{z2}=\frac{U_xU_y}{U_k}+U_y\delta\alpha_{3n}-U_k\delta\alpha_{3s}\delta\alpha_{4s}-U_k\delta\alpha_{3s}\delta\alpha_{4n}.$$

*In this specific network we provide for turning off the multiplying device M when one of its multipliers is small or equal to zero. The device is turned off by introducing a small dead band between M and the adder Σ_3.

From this we obtain

$$\delta U_z = -\delta\alpha_{3s}\delta\alpha_{4s} - \delta\alpha_{3s}\,\delta\alpha_{4n} + \frac{U_y}{U_{z\,max}}\delta\alpha_{sn}. \tag{4}$$

The basic error must be determined by the third term in this expression; i.e., it must be determined by the nonidentity error. Therefore, there is no purpose in increasing the accuracy with which the transfer coefficients are generated. It is evident that $\delta\alpha_s$ is conveniently chosen in accordance with the accuracy requirements for alternating current. Since these requirements are usually appreciably lower than those which apply to the static accuracy, it follows that the multiplying devices in each of the channels can be simplified. In networks with binary counters, it is possible to reduce the number of stages; in networks with potentiometers it is possible to replace an amplifier with a polarized relay, etc. Just as in coarse-fine systems, it is possible to make use of stepfinders.

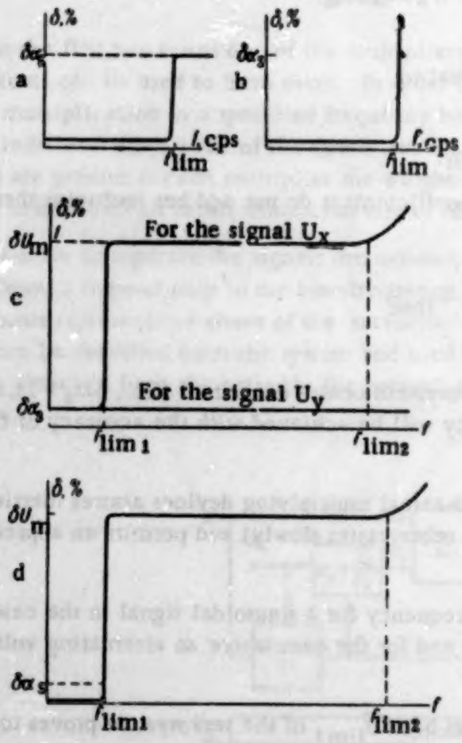

Fig. 6.

When one electromechanical and one high-frequency multiplying device are included in the network shown in Fig. 5 the multiplication is performed according to the formula $U_z = \frac{U_y\tilde{U}_x + U_y\bar{\bar{U}}_x}{U_k}$. The static accuracy and the dynamic error with respect to one of the inputs will be determined by the electromechanical device. As the frequency of the signal corresponding to the other multipliers increases while the first one remains constant, or in the case where an alternating voltage is multiplied by itself, the electromechanical network will operate first, followed by the high-frequency network; this procedure is the same as for a multiplying device with a nonlinear transfer function. The graph showing the variation of the error for this case is shown in Fig. 6c.

Two shortcomings of such a two-channel unit with frequency division of the signal are: in only one of the multipliers are the increased requirements imposed on the electromechanical channel and the low accuracy of multiplying the high-frequency voltage $\tilde{U}_x$ by U_y.

For the case where three channels are used in the network, all of the favorable properties of the first two variants are used.

The accuracy of multiplication by zero and by small voltages becomes very high, since the multiplying device M does not operate under these conditions. When one of the multipliers is a constant quantity, the graph for the variation of the error as a function of the frequency of the other multiplier is of the same shape as it is for a network with two electromechanical multiplying devices (Fig. 6a). The graphs for the variation of the error in the case of squaring an alternating voltage is shown in Fig. 6d.

3. A VARIANT OF A THREE-CHANNEL MULTIPLYING DEVICE WHICH USES REVERSIVE STEPFINDERS

The block diagram of the unit is shown in Fig. 7. The electromechanical multiplying devices in the low-frequency channel are designed using reversive stepfinders of the type "RShI-50/4". In each finder we use two tracks, each of which contains 50 commutator segments. The transfer coefficients for the dividers $D_1 - D_4$ can vary discretely in steps of 0.02 of the maximum value. The mismatch voltages for the servosystems $U_x - \bar{\bar{U}}_x$ and $U_y - \bar{\bar{U}}_y$ are applied to the windings of polarized relays of the type "RP-5" through two serially connected cathode

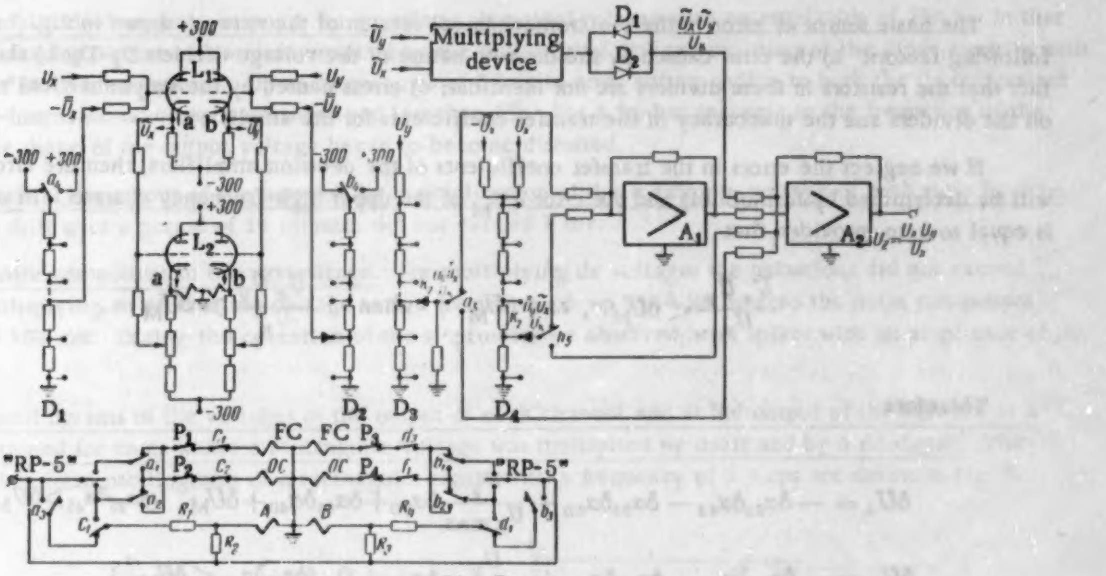

Fig. 7.

followers. When the error in one of the servosystems exceeds the quantization error, the armature of a "RP-5" is attracted to one of its contacts, and a voltage of 48 v begins to be applied to one of the electromagnets of the stepfinder. The windings of the stepfinder coils are connected through self-breaking contacts which disconnect the voltage at the instant the armature is attracted to the electromagnet; therefore, the brush of the stepfinder can be switched from commutator segment to commutator segment until the armature of the "RP-5" relay returns to the intermediate position.

The electromagnetic relays A and B (of the type "RS-13") assure operation of the network in four quadrants. When the input voltages U_x and U_y are positive the windings of these relays are de-energized; therefore, negative reference voltages are applied to the dividers D_1 and D_2, and the components $U_y\overline{U}_x/U_k$ and $\overline{U}_y\tilde{U}_x/U_k$ of the product are applied to the input of amplifier 2 through a sign-inversion amplifier 1. If, for example, the signal U_x changes sign, then as a result of the attraction of the polarized relay armature in the left side of the network to the lower contact the brushes of the stepfinder "ShI-1" are initially shifted to the zero position. Under these conditions, the contacts C_1 and C_2 which are controlled by rotor of the stepfinder are switched, the reverse-count winding is disconnected, and a voltage is applied to the winding of relay A through the resistance R_1. Relay A operates, the sign of the reference voltage across the divider D_1 changes, and the voltage from the lower contact of the polarized relay is applied to the forward-count winding FC. After the finder brush has been shifted one commutator segment in the forward direction the contact C_1 is switched to its original position, but the relay A has had time to be blocked through its contact a_3. In order to insure that the relay does not release during the time required to switch the contact C_1, its release time is somewhat retarded by a magnetizing current flowing through the resistor R_2. Then the relay A remains closed until the finder brush again returns to the zero position and the contact C_1 is switched.

The rule governing the signs is followed during the multiplication process by switching the component $\overline{U}_xU_y/U_k$ to the input of amplifier 2 by means of the contact a_5; the sign-converter 1 is bypassed. The second system operates analogously.

As a high-frequency channel we used diode squaring devices taken from the multiplying unit in the "EMU-5" simulator. The variable components of the multipliers U_x and U_y are taken from the outputs of cathode followers based on the tube L_1 (a "6N3P" tube; its characteristics are: $E_f = 6.3$ v, $I_f = 0.35$ amp, $E_\rho = 150$ v, $I_\rho = 7.7$ ma, $R_k = 240$ ohm, $g_m = 4900$ μmho, $r_\rho = 7.5$ k, $P_\rho = 1.5$ w).

In order that the multiplying unit in the upper channel should not create an additional error in those cases when there is no high-frequency component in one of the input voltages, a circuit consisting of two parallel silicon diodes was connected to its output. These diodes begin to pass a voltage which is above a threshold of 0.4-0.5 v; therefore if $\tilde{U}_x$ or $\tilde{U}_y$ are equal to zero an error voltage lying within these limits is not passed. When $\tilde{U}_x\tilde{U}_y/U_k \neq 0$, the accuracy of the result remains practically unchanged.

The basic source of error in the electromechanical section of the network shown in Fig. 7 consists of the following factors: a) the error caused by the discrete nature of the voltage dividers D_1-D_4; b) the error due to the fact that the resistors in these dividers are not identical; c) errors caused by the unsymmetrical nature of the load on the dividers and the inaccuracy of the transfer coefficients for the amplifiers.

If we neglect the errors in the transfer coefficients of the decision amplifiers, then the error of the network will be determined by formula (4) and the error δU_M of the upper high-frequency channel. In our case, the latter is equal to zero, provided that

$$\frac{U_x U_y}{U_н} < \delta U_M \text{ , and } \delta U_M \text{ , when } \frac{U_x U_y}{U_н} > \delta U_M \text{ .}$$

Therefore

$$\delta U_z = -\delta\alpha_{3s}\delta\alpha_{4s} - \delta\alpha_{3s}\delta\alpha_{4n} + \frac{U_y}{U_{z\,max}}\delta\alpha_{3n} + \delta\alpha_{3s}\delta\alpha_{4n} + \delta U_M \quad (\delta\alpha_{3s}\delta\alpha_{4s} > \delta U_M) \tag{5}$$

$$\delta U_z = -\delta\alpha_{3s}\delta\alpha_{4s} - \delta\alpha_{3s}\delta\alpha_{4n} + \frac{U_y}{U_{z\,max}}\delta\alpha_{3n} + 0 \quad (\delta\alpha_{3s}\delta\alpha_{4s} < \delta U_M).$$

The errors due to the discrete nature of the voltage dividers D_1 and D_2 were made equal to 2% in our network. The resistors were manufactured with a tolerance of 0.1%. Under these conditions the maximum possible value of the static error of the multiplying unit did not exceed 0.15%; the accuracy for multiplying high-frequency signals was not less than 1.15%.

4. THE RESULTS OF AN EXPERIMENTAL VERIFICATION OF THE MULTIPLYING UNIT

In the process of performing an experimental study of the new multiplying unit, we obtained the following characteristics.

1. The static error for input voltages of different signs. The voltages were measured by the compensation method using a normal battery element as a standard source. The results of our tests when the network operated in the first quadrant are cited in the table below. The table fixes the error voltages in volts which were computed as the differences between the calculated and measured values of the output voltage. When the network operated in the other quadrants the maximum error also did not exceed 0.1%.

U_y	U_x: 0	10	20	30	40	50	60	70	80	90	100
0	0	0	0	0	0	0	0	0	0	0	0
10	0	−0.01	−0.03	0	0.01	0.01	0.05	0.02	0.03	−0.01	0
20	0	−0.02	−0.02	0.01	−0.01	−0.01	−0.07	−0.06	−0.04	−0.02	0.01
30	0	0.01	0.01	0	0.01	0.05	0.06	0.03	0	−0.01	−0.03
40	0	0	−0.04	−0.08	−0.04	−0.03	−0.01	−0.02	0.02	0.03	0.05
50	0	0.04	0.06	0.02	0.03	0.01	0.01	0	−0.02	−0.03	−0.01
60	0	0.01	0.03	0.05	0.02		−0.01	−0.03	−0.06	−0.03	−0.02
70	0	−0.03	−0.01	0	0.01	0.01	0.06	0.05	0.03	0	−0.01
80	0	−0.01	−0.07	−0.04	−0.03	−0.01	0	0.01	0.03	0.04	0.05
90	0	0.02	0.04	0.03	0.01	0.01	0.02	−0.01	−0.01	−0.04	−0.03
100	0	0.01	0.02	0	−0.01	−0.03	−0.07	−0.06	−0.02	0	0.01

2. The amplitude-frequency response with respect to each of the inputs for a constant second multiplier. The frequency response did not drop off within the limits of the pass band of the "EMU-8"-simulator decision amplifiers used in our mockup; i.e., it remained flat in the frequency range 0 to 10 kc for an output voltage amplitude equal to 100 v. The error in the indicated frequency range did not exceed 1%.

3. The amplitude-frequency response for squaring a sinusoidal voltage with an amplitude of 100 v. In that case, the pass band for the network was determined by the multiplying device consisting of the diode squaring unit taken from the "EMU-5"-simulator. Up to frequencies of 5 kc the error corresponding to both the dc component and the second-harmonic ac component remained less than 1%. For a fruther increase in the frequency of the input signals the shape of the output voltage began to become distorted.

4. The drift at the output of the network for multiplication of the maximum voltage of both signs by zero. The maximum drift over a period of 10 minutes did not exceed 1 mv.

5. The noise component in the out voltage. For multiplying dc voltages the pulsations did not exceed 30 mv. For multiplying an alternating voltage with a frequency of up to 10 kc by zero the noise component did not exceed 100 mv. During the operation of the stepfinders we observed brief spikes with an amplitude of up to 250 mv.

6. The oscillograms of the voltages at the output of each channel and at the output of the network as a whole were obtained for cases where a rectangular voltage was multiplied by itself and by a dc signal. The photographs taken from oscillograms of a rectangular signal with a frequency of 0.3 cps are shown in Fig. 8.

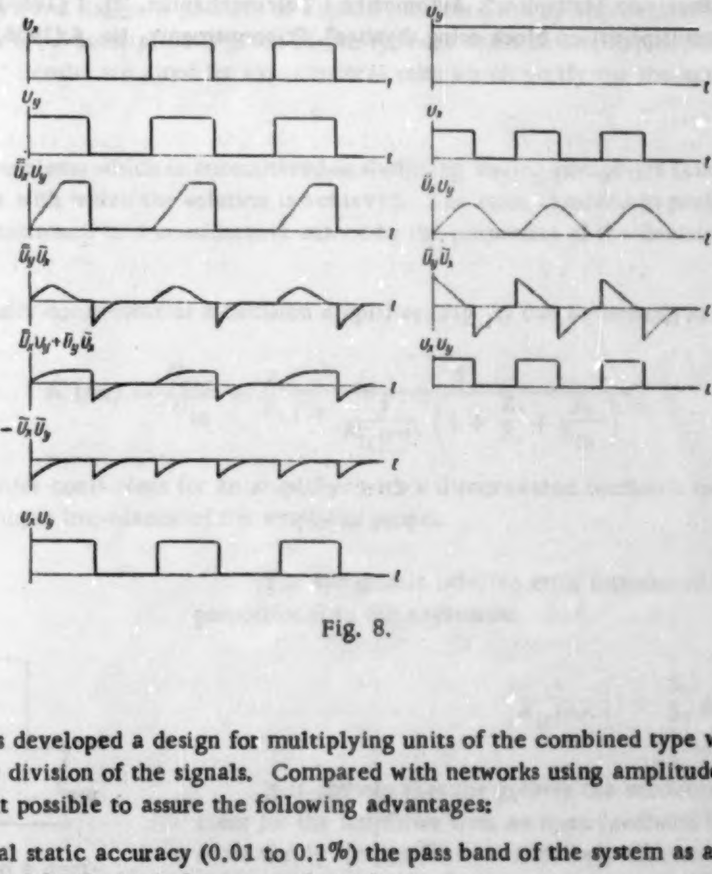

Fig. 8.

CONCLUSION

1. The author has developed a design for multiplying units of the combined type with three parallel channels and frequency division of the signals. Compared with networks using amplitude division of the signals, the new design makes it possible to assure the following advantages:

a) For an identical static accuracy (0.01 to 0.1%) the pass band of the system as a whole is extended up to the width of the pass band for the high-frequency channel.

b) It is possible to subdivide the network into individual multiplying devices without changing their circuits; here the engineering characteristics of the system are altered as a function of the number of operating channels.

c) It is possible to perform multiplication operations (even though the accuracy is lower) when one or two channels become inoperative; and this assures high reliability for the proposed unit.

d) The over-all network is distinguished by its comparative simplicity, since notwithstanding the presence of three multiplying devices each of them is subjected to less rigorous requirements than the requirements which apply when one of them is operating independently. This makes it possible to simplify the circuit for each of the channels.

2. The author has developed a variant of a three-channel network using stepfinders; experimental verification of this network proved the advantage of the new design for multiplying units.

SUMMARY

The survey of combined type of multiplying devices with amplitude-division channels is given. A new structure of a multiplying device with frequency division of input signals is proposed and its properties are considered. The data of the experimental check of a reversive step selector circuit are given.

LITERATURE CITED

1. A. A. Maslov, "Multiplying units (survey)", Avtomatika i Telemekhanika, 21, 10 (1960). *
2. A. A. Fel'dbaum and L. N. Fitsner, An Electrical Multiplying Device, Patent Application Number 104137, September 3, 1952.
3. L. N. Fitsner, "A Precision Multiplier", Avtomatika i Telemekhanika, 20, 1 (1959).
4. L. N. Fitsner, "A multiplication block using thyrites", Priborostroenie, No. 4 (1956).

* See English translation.

WIDE-BAND DECISION (OPERATIONAL) AMPLIFIERS

D. E. Polonnikov
(Moscow)

Translated from Avtomatika i Telemekhanika, Vol. 21, No. 12,
pp. 1613-1622, December, 1960
Original article submitted May 7, 1960

A principle is proposed for designing decision amplifiers; this principle is based on the sequential switching of stages and makes it possible to expand the pass band by approximately two orders of magnitude. A brief description is given of two types of decision amplifiers designed according to this principle. Results are cited for experimental tests which verify the theoretical propositions.

One of the basic problems which is encountered in designing analog computers (electronic simulators) is that of lowering the error with which the solution is achieved. The error involved in performing individual mathematical operations is determined to a considerable extent by the properties of the decision amplifier (in particular, by its pass band).

The complex transfer coefficient of a decision amplifier (Fig. 1) can be written as follows:

$$K(j\omega) = \frac{U_{\text{out}}}{U_{\text{in}}} = \frac{Z_2}{Z_1} \frac{1}{1 + \frac{1}{K_{\text{tr}}(j\omega)} \left(1 + \frac{Z_2}{Z_1} + \frac{Z_2}{Z_{\text{in}}}\right)}, \tag{1}$$

where $K_{tr}(j\omega)$ is the transfer coefficient for an amplifier with a disconnected feedback loop, $Z_{in} = R_{in}/(1 + j\omega R_{in}C_{in})$ is the total input impedance of the amplifier proper.

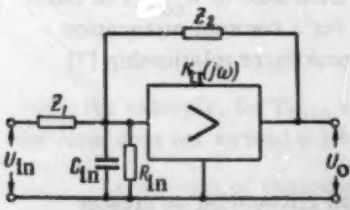

Fig. 1. Block diagram of a decision (operational) amplifier.

The systematic relative error introduced by the amplifier is proportional to the expression

$$\left| \frac{1}{K_{\text{tr}}(j\omega)} \left(1 + \frac{Z_2}{Z_1} + \frac{Z_2}{Z_{\text{in}}}\right) \right|.$$

It is obvious that the greater the modulus of the transfer coefficient for the amplifier with an open feedback loop $|K_{tr}(j\omega)|$, and the more slowly this coefficient decreases with an increase in frequency, the smaller the error introduced by the amplifier and the more slowly this error increases with an increase in frequency.

In the majority of problems encountered in practice it is sufficient to have a high accuracy for the solution of the frequency range from zero to one kilocycle. It is not difficult to assure such a pass band for an amplifier with an open feedback loop. However, if we take into account the requirement that stability must be assured when the feedback loop is closed, the problem is seriously complicated since the decision amplifier always contains several stages of amplification and its dynamic properties are described by a high order of transfer coefficient with a large $|K_{tr}(0)|$.

In the analysis below we shall describe the principle for designing decision amplifiers which consists of the sequential connection of stages. Compared to the conventional single-circuit network, such a principle makes it

possible (other conditions being equal) substantially to increase the stability margin and to expand the pass bands for the amplifier. As an example of the practical application of this principle we describe two decision amplifier networks.

1. THE PRINCIPLE OF SEQUENTIAL SWITCHING OF STAGES

Figure 2 shows the block diagram of a single-channel four-stage amplifier with an open feedback loop for high frequencies.* Its transfer coefficient can be written as follows:

$$K_{tr1} = \frac{U_{out}}{U_{in}} = \frac{K_1 K_2 K_3 K_{out}}{(1+j\omega T_1)(1+j\omega T_2)(1+j\omega T_3)(1+j\omega T_{out})}, \quad (2)$$

where K_1, K_2, K_3, K_{out} are the gains of the corresponding stages for dc operation; T_1, T_2, T_3, T_{out} are the equivalent time constants which are equal to the product of the output resistance of the corresponding stage and the total capacitance connected to its output (this includes the mounting capacitance and the input capacitance of the preceding stage). In analyzing the amplifier stability it is also necessary to take into account the transfer coefficient for the feedback circuits. But in decision amplifiers the feedback factor varies over wide limits, and therefore for convenience we shall assume that it is equal to zero; this will in no way effect the essence of the comparative analysis cited below.

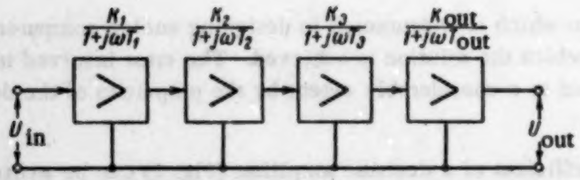

Fig. 2. Block diagram for a single-channel four-stage amplifier without feedback at high frequencies.

In order to ensure absolute stability** for the amplifier when the feedback loop is closed it is necessary that the phase (the argument) of the transfer coefficient for the open-loop circuit not exceed π for all frequencies from zero to the cutoff frequency ω_{co}. The cutoff frequency*** is defined as the frequency at which the modulus of the transfer coefficient is equal to unity; i.e., $|K_{tr}(j\omega_{co})| = 1$.

Since a dc decision amplifier as a rule is treated as a minimum-phase system, the stability requirement means that the attenuation of $|K_{tr}(j\omega)|$ must not exceed 40 db/decade in the range from zero to ω_{co}. The latter condition follows from the properties of logarithmic amplitude-frequency responses. For a constant attenuation γ expressed in decibels per frequency decade, the phase φ is determined from the approximate relationship [7]

$$\varphi = -\frac{\pi\gamma}{40}.$$

In practice a definite stability margin is required; therefore we usually choose an attenuation no greater than 33 db/decade for which the phase does not exceed 150°. In order to guarantee this condition in a four-stage amplifier it is necessary that at least two stages have time constants that are so small that the phase shift introduced by them is negligibly small right up to the cutoff frequency. It we assume that the entire circuit of the

*It is assumed that one of the stages does not invert the phase.

**It is impractical to construct a conditionally stable system, since when the magnitude and form of the transfer coefficient vary, the feedback circuit of the system may prove to be unstable.

***The cutoff frequency also characterizes the pass band of the amplifier in the closed-loop state. For a transfer coefficient equal to unity the upper boundary of the pass band (at this boundary the gain decreases by 3 db) is equal to $0.3\,\omega_{co}$.

open-loop amplifier produces a phase shift no greater than 140° (an attenuation of 31 db/decade), then 10° is left for the two stages with minimal time constants. On this basis it is possible to write the conditions for a single-channel amplifier which permit us to relate the minimal time constant, the cutoff frequency and the frequency ω_{h1} at which the attenuation of $|K_{tr1}(j\omega)|$ * begins (Fig. 3):

$$\frac{\omega_{co.1}}{\omega_{h1}} = 10^{\frac{20\log|K_{tr}(0)|}{\gamma}}, \tag{3}$$

$$\omega_{co.1}T_{min} \approx \frac{\tan 10°}{2} \approx 0{,}1, \tag{4}$$

where γ is the attenuation coefficient expressed in decibels per decade (in our case 31 db/decade). Note that formula (3) is valid in the general case, irrespective of the number of channels.

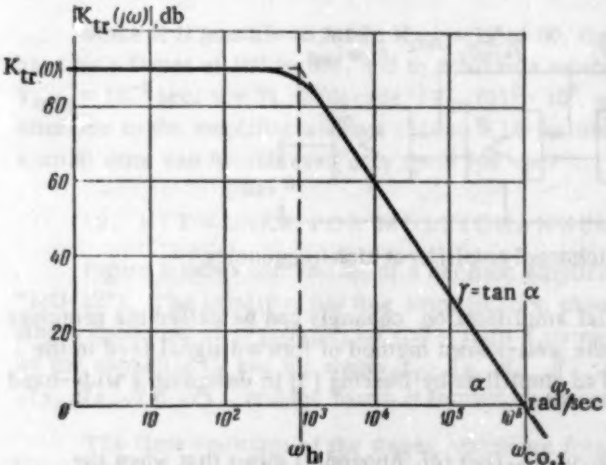

Fig. 3. Logarithmic amplitude-frequency response for the amplifier.

If the allowable error with which the operation can be performed in a definite frequency band is specified (this means that $|K_{tr1}(0)|$ and ω_{h1} are specified), then it follows from (3) and (4) that we can find the cutoff frequency and the maximum allowable T_{min}. For example, if we assume that an error of 0.1% is allowable in the range from 0 to 1000 cps, then it is necessary to choose $|K_{tr1}(0)| \leq 10^5$**, $\omega_{h1} = 2\pi \cdot 10^3$ rad/sec, $\omega_{co1} = 10^7$ rad/sec and $T_{min} \leq 10^{-8}$ sec. It is extremely difficult to ensure such a small time constant for an amplifying stage in a decision amplifier. To achieve this, it would be necessary to increase the number of stages and sharply to increase the power requirements. Therefore, in conventional single-loop systems it is necessary to be satisfied either with a narrower frequency band or to increase the allowable error. If we assume that the quantity T_{min} is specified, then we find ω_{co1} and ω_{h1} by transforming (3) and (4):

$$\omega_{co1} \approx \frac{1}{10\,T_{min}}, \tag{5}$$

$$\omega_{h1} \approx \frac{1}{10T_{min}}\,10^{-\frac{20\log K_{tr}(0)}{\gamma}}. \tag{6}$$

For example, for $T_{min} = 0.1$ μsec and $\gamma = 31$ db/decade the highest frequency $f_{h1} = \omega_{h1}/2\pi$, for which the error does not exceed 0.1% is only 100 cps.

The problem of expanding the pass band and assuring adequate stability is appreciably simplified in the network shown in Fig. 4. The input signal from the summing point is applied not only to the first stage but to all subsequent stages (the first stage provides for the possibility of summing the signals in such a way that the feedback with respect to each channel is negative). The transfer coefficient for such a four-channel network is given by the following formula:

---

* Here and throughout, the second subscript is used to denote the number of channels (circuits) in the network which is characterized by the specified quantity. For example, K_{tr1} is the transfer coefficient for a single-channel amplifier.

** The quantity $K_{tr1}(0)$ is computed for the case of operation as an adder for six inputs with the transfer coefficients 10, 5, 5.1, 1.1 and with a capacitance of up to 500 μμf at the summing point.

$$K_{tr4}(j\omega) = \left\{\left[\left(\frac{K_1}{1+j\omega T_1}+1\right)\frac{K_2}{1+j\omega T_2}+1\right]\frac{K_3}{1+j\omega T_3}+1\right\}\frac{K_{out}}{1+j\omega T_{out}}. \tag{7}$$

If we choose

$$T_1 \gg T_2 \gg T_3 \gg T_{out}, \tag{8}$$

then as the frequency increases, the attenuation $|K_{tr4}(j\omega)|$ will first be caused by the time constant T_1 of the first stage; under these conditions the remaining time constants have practically no effect on the attenuation. The signal is amplified by all the stages. The auxiliary channels do not have any appreciable effect. But when the gain of the first stage becomes less than unity at the frequency $\omega_1 > K_1/T_1$, the signal will pass chiefly through the second channel to the second stage. The first stage together with its time constant T_1 will be switched off. At frequencies exceeding $\omega_3 > K_3/T_3$ the third stage is switched off. The signal will then pass directly to the output stage. Thus, at higher frequencies, for all practical purposes only the time constants T_{out} remains.

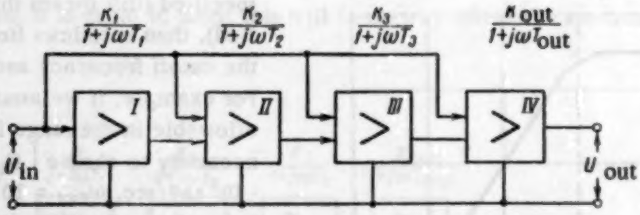

Fig. 4. Block diagram for a multichannel amplifier at high frequencies.

Such a principle for designing a network with parallel amplification channels can be called the principle of sequential switching of stages. It is a development of the well-known method of forward signal feed in the theory of automatic control; this method was first applied to amplifiers by Deering [3] in designing a wide-band inverter (the network had two channels).

An analysis of the expression for the transfer coefficient $K_{tr4}(j\omega)$ (cf. Appendix) shows that when the following conditions are satisfied:

$$T_1 = K_1T_2,\ \ T_2 = K_2T_3,\ \ T_3 = K_3T_{out}, \tag{9}$$

the attenuation at any frequencies does not exceed 20 db/decade, and the phase lag does not exceed $\pi/2$; i.e., there is a very large stability margin. In that case ω_{co4} and ω_{h4} are correspondingly equal to

$$\omega_{co.4} = \frac{K_{out}}{T_{out}} = \frac{K_{out}}{T_{min}}, \tag{10}$$

$$\omega_{h4} = \frac{1}{T_1} = \frac{1}{K_1K_2K_3T_{min}}. \tag{11}$$

It can be shown (cf. Appendix) that if we specify the allowed phase shift or attenuation γ, then the time constants are approximately chosen from the following conditions:

$$T_1 = \frac{K_1T_2}{\sqrt[3]{K_1K_2K_3K_{out}}}\,10^{\frac{20\log|K_{tr}(0)|}{3\gamma}}, \tag{12}$$

$$T_2 = \frac{K_2T_3}{\sqrt[3]{K_1K_2K_3K_{out}}}\,10^{\frac{20\log|K_{tr}(0)|}{3\gamma}}, \tag{13}$$

$$T_3 = \frac{K_3 T_{out}}{\sqrt[3]{K_1 K_2 K_3 K_{out}}} 10^{\frac{20 \log |K_{tr}(0)|}{3\gamma}} \tag{14}$$

Under these conditions the frequency at which the attenuation of K_{tr} begins can be increased appreciably. On the basis of (3) and (10),

$$\omega_{h4} = \omega_{co.4} 10^{\frac{-20 \log |K_{tr}(0)|}{\gamma}} = \frac{K_{out}}{T_{min}} 10^{\frac{-20 \log |K_{tr}(0)|}{\gamma}} \tag{15}$$

The advantages offered by the principle of sequential switching of stages is clearly characterized by the ratios $\omega_{co4}/\omega_{co1}$ and ω_{h4}/ω_{h1} for equal T_{min} and γ:

$$\frac{\omega_{co.4}}{\omega_{co.1}} = \frac{\omega_{h4}}{\omega_{h1}} \approx 10 K_{out} \tag{16}$$

Since it is possible to assure K_{out} = 10 to 50, the proposed principle makes it possible to expand the pass band by a factor of 100 to 500, and to achieve a substantial lowering of the dynamic error. For example, for $T_{min} = 10^{-7}$ sec, γ = 31 db/decade, $|K_{tr4}(0)| = 10^5$, and K_{out} = 20, we have $\omega_{h4} = 1.25 \cdot 10^6$ rad/sec; i.e., the error due to the amplifier will not exceed 0.1% for frequencies right up to 20 kc (for a conventional network such a small error can be achieved only up to 100 cps).

2. NETWORKS FOR MULTICHANNEL WIDE-BAND DECISION AMPLIFIERS

Figure 5 shows one variant of a decision amplifier network with sequential switching of the stages (type "MU-12"). The amplifier has four amplification channels. One is formed by the MDM* amplifier and the stages consisting of $Л_3, Л_4, Л_5$ ($Л_4, Л_5$ form a series-balanced output cathode follower); the second is formed by the capacitor C_1 and the stages $Л'_1$, $Л_2$, $Л_3$, $Л_4$, $Л_5$,; the third is formed by the capacitor C_3 and the stages $Л_2$, $Л_3$, $Л_4$, $Л_5$, and the fourth is formed by the capacitors C_3 and C_5, and the stages $Л_3$, $Л_4$, $Л_5$.

The time constants of the stages, which are formed by the output resistors and capacitors of the corresponding stages (and for the MDM channel the time constant is equal to the time constant of the demodulator filter), are chosen in such a way that as the frequency increases the MDM channel is switched off first, followed by the stage $Л_2$ under these conditions the signal is amplified solely by the stage $Л_3$.

The network shown in Fig. 5 differs somewhat from the block diagram shown in Fig. 4 in that the network in Fig. 5 does not have a channel which would apply the signal directly to the output stage. Because of this the phase shift at high frequencies is greater than that produced by the ideal network due to the presence of the time constant of the stage consisting of $Л_3$. However, it is possible to make this time constant so small (due to the small input capacitance of the output cathode follower) that it begins to have an appreciable effect only at frequencies above the cutoff frequency; thus, it has no effect on stability.

In testing a mock-up of an amplifier designed according to the network shown in Fig. 5 we were able to obtain the following results.

1. The pass band (with respect to 3 db attenuation) for a transfer coefficient equal to unity, U_{out} = 10 v, and 50 $\mu\mu$f capacitors at the summing point and at the output is equal to 7 Mc.** When the capacitors are increased to 500 $\mu\mu$f the pass band is reduced to 0.5 Mc. For the same parameters with the exception of U_{out} = = 100 v we obtained pass band values of 1 Mc and 100 kc, respectively.

2. The noise level at the output for a transfer coefficient equal to unity was 1 mv (in amplitude); for a transfer coefficient of a thousand it was equal to 0.5 v.

*The MDM amplifier consists of a modulator, a three-stage ac amplifier, and a demodulator.
**The feedback resistance and the 1 Meg input resistance were shunted by 22 $\mu\mu$f capacitors.

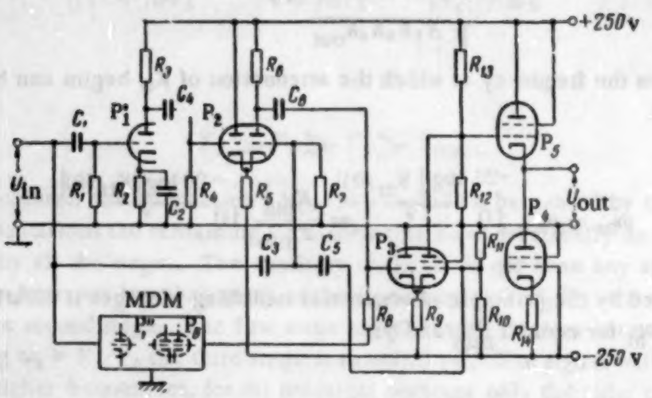

Fig. 5. Network for a multichannel decision amplifier ("MU-12") with a pass band of 7 Mc.

3. The null drift over a period of 8 hours was 50 μv.

Figure 6 shows a different variant of the network in which the principle of sequential switching of stages is used only partially. The signal from the summing point is applied to only the first stage ($Л_2'$, $Л_8$) in addition to the MDM channel ($Л_1$), this first stage has a wide pass band. The first stage is followed by two channels; one connects the stage to $Л_2'$, , and the second passes the signal through C_8 directly to the summing stage ($Л_8$) As the frequency increases, the second stage is switched off. The time constant for the first stage is lowered to $8 \cdot 10^{-8}$ sec due to the use of a cathode follower; the output time constant of the summing stage is also small and equals $3 \cdot 10^{-7}$ sec. Because of this it was possible to achieve an adequate stability margin for a wide pass band (cf. Fig. 8).

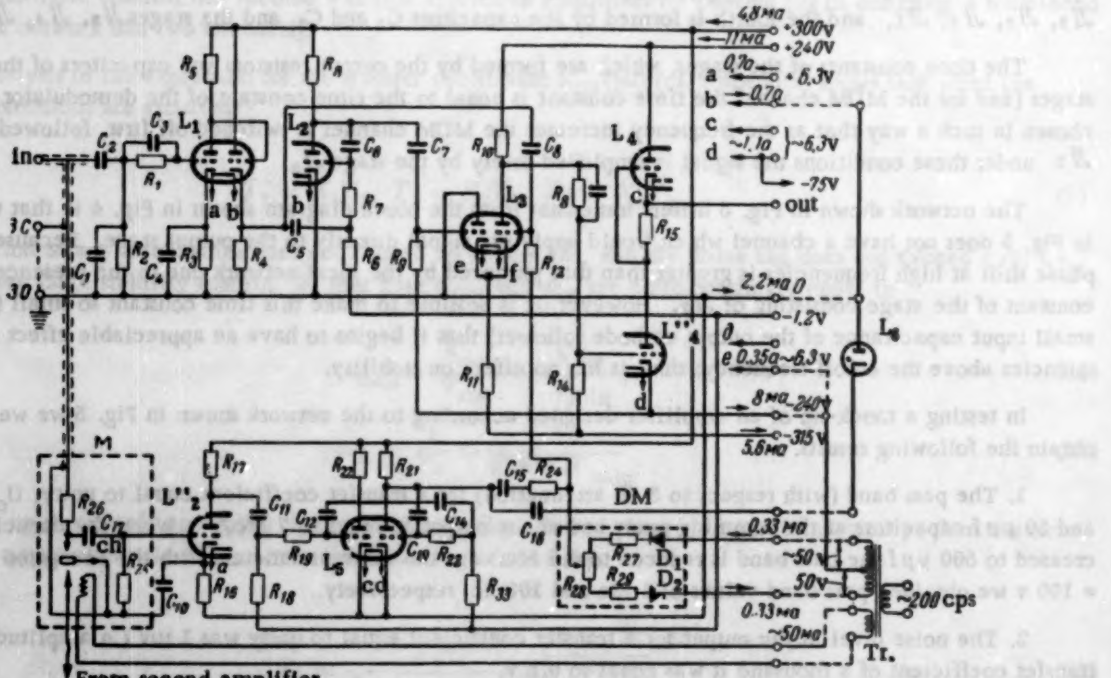

Fig. 6. Network for a three-channel amplifier ("TU-10") with a pass band of 1 Mc. Ch. is a "chopper", M is a modulator, DM is a demodulator.

Note that an amplifier which is encompassed with 100% feedback is unstable. But, when the transfer coefficient for the feedback loop is taken into account (the feedback resistance and the capacitance of the summing

point with respect to ground) the amplifier has an appreciable stability margin. In particular, it proved to be permissible for a capacitor up to 0.1 μf to be connected to the summing point and for a capacitor of up to 0.5 μf to be connected at the output. Thus, the capacitance of the summing point (which usually worsens stability) is used as an element in a useful corrective network in this case.

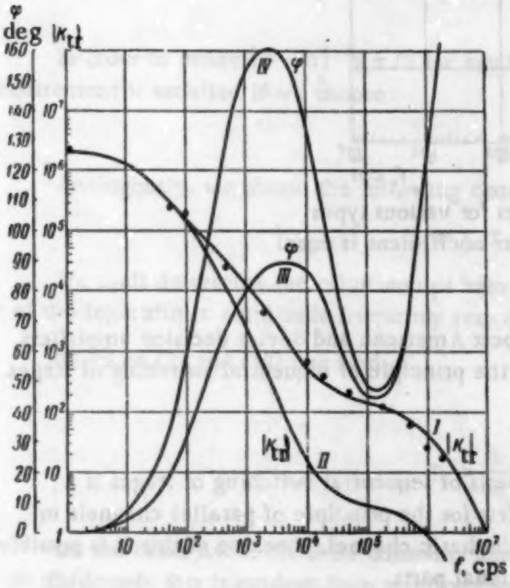

Fig. 7. The computed amplitude-frequency and phase-frequency responses for a "TU-10" amplifier with an open feedback loop. Curves I and III were taken with no corrective network. Curves II and IV were taken for the case when a corrective network was present. The points represent the experimental results.

When the ac transfer coefficient of the feedback loop must be increased (for example, in the case of a limiter or integrator), the connection of a corrective network in series with the input is provided for. Under these conditions the amplitude-frequency and phase-frequency responses acquire the form shown in Fig. 7 (curves II and IV).

The decoupling capacitors C_2, C_5, C_7, C_8 and resistors R_2, R_6, R_9, R_{12} were chosen on the basis of the requirements for assuring stability at low frequencies and satisfying the conditions governing joint operation with the MDM channel. The remaining elements in the network were chosen in such a way as to assure operation of the stages in the linear region with low plate current requirements.

Below we shall cite certain results obtained by testing a three-channel amplifier ("TU-10") which was designed according to the network shown in Fig. 6.

1. The null drift when the unit operated as a scale amplifier normalized to the input (for K = 10) was equal to ± 30 μv over a period of 8 hours.

2. The null drift when the device operated as an integrator for RC = 1 was 15 mv during 1000 sec (or 1.5 mv per 100 sec).

3. The noise at the output for a transfer coefficient equal to unity was 5 mv (the amplitude value).

4. The linear range was equal to ± 145 v for a 10 kohm load, ± 115 v for a 5 kohm load, and ± 100 v for a 2 kohm load.

5. The gain for the amplifier when the feedback loop was open was equal to $(1\text{-}2) \cdot 10^6$ for dc, $8 \cdot 10^4$ at a frequency of 200 cps, 600 at a frequency of 10 kc, and 150 at a frequency of 100 kc (cf. Fig. 7).

6. The frequency at which the amplifier with a transfer coefficient of unity has an error of 0.1% was equal to 1 kc; at 10 kc and 1 Mc the errors were 1% and 30%, respectively (for $U_{out} \le 10$ v).

The pass band (30% drop in the amplitude-frequency response) for $U_{out} = 100$ v, a load of 10 kohm, and a shunt capacitance of 600 $\mu\mu$f was 65 kc; for a shunt capacitance of 2000 $\mu\mu$f it was 25 kc, and for a capacitance of 0.01 μf it was 4 kc.

7. The time required to restore the working mode after amplifier overload was no greater than 3 sec.

8. The maximum allowable capacitance at the amplifier output (for a 700 $\mu\mu$f capacitance at the summing point) was no less than 0.25 μf (when the device operated as an amplifier or as an integrator). The maximum allowable capacitance at the summing point (for an output capacitance of 1000 $\mu\mu$f) was no less than 0.25 μf. Under these conditions the pulsations increased to 1 v. For the case of operation as a limiter or operation with variable resistor in the feedback loop, the maximum allowable values of capacitance at the summing point were reduced to 500 $\mu\mu$f for an output capacitance of up to 0.03 μf, or down to 1500 $\mu\mu$f for an output capacitance of up to 0.015 μf.*

*All of the data cited here were obtained when the last two stages (which are the main users of the power supply) were supplied from unstabilized rectifiers with pulsations of 3 v.

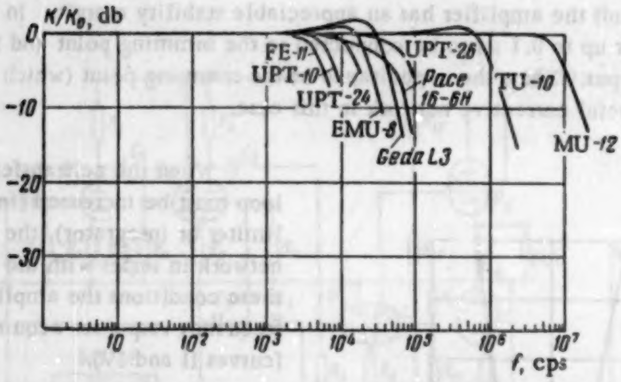

Fig. 8. Amplitude-frequency responses for various types of decision amplifiers when the transfer coefficient is equal to unity.

Figure 8 compares the amplitude-frequency responses of the best American and Soviet decision amplifiers and shows the advantages of the new amplifiers which are based on the principle of sequential switching of stages.

CONCLUSIONS

1. The proposed principle for designing of amplifiers on the basis of sequential switching of stages is a further development of the principle of parallel channels [2]; it differs for the principle of parallel channels in that the auxiliary channels are formed by elements that enter into the basic channel. Because of this it is possible to design a multichannel network with practically no need for additional parts.

2. The design of decision amplifiers according to the principle of sequential switching of stages permits us (other conditions being equal) to expand the pass band by at least two orders of magnitude while retaining the previous stability margin, or to increase the stability margin for the same pass band.

3. We have derived the conditions (12) and (14) for selecting the network time constants which assure the specified attenuation of the amplitude-frequency response; we have also established the relationship (6), (15) between the frequency range ω_h in which the specified accuracy is assured and the minimal time constant for the system.

4. Based on the principle proposed in this paper we designed two decision amplifier networks (amplifiers of the type "TU-10" and "MU-12") whose application in electronic simulators permits us substantially to lower the static and dynamic error components caused by the null drift and pass band in a decision amplifier.

APPENDIX

Analysis of the Expressions for the Transfer Coefficient of a Multiloop Network

We shall find the conditions for which the phase lag introduced by the amplifier (cf. Fig. 4) does not exceed $\pi/2$ at any frequency. For simplicity we shall study a three-stage amplifier without an output stage. The transfer coefficient will be written as follows for this case:

$$K_{tr3} = \left[\left(\frac{K_1}{1+j\omega T_1}+1\right)\frac{K_2}{1+j\omega T_2}+1\right]\frac{K_3}{1+j\omega T_3}+1. \tag{17}$$

Separating the real an imaginary parts, we find

$$\tan\varphi = -\frac{d}{b}\,\omega, \tag{18}$$

where

$$d = K_1K_2K_3(T_1 + T_2 + T_3) + K_2K_3(T_2 + T_3) + K_3T_3 -$$
$$- \omega^2 K_1K_2K_3T_1T_2T_3 + \omega^2 K_2K_3T_1^2(T_2 + T_3) + \omega^2 K_3T_3(T_1^2 + T_2^2) + \omega^4 K_3T_1^2T_2^2T_3, \tag{19}$$
$$b = K_1K_2K_3 + K_2K_3 + K_3 + 1 - \omega^2 K_1K_2K_3(T_1T_2 + T_1T_3 + T_2T_3) +$$
$$+ \omega^2 K_2K_3(T_1^2 - T_2T_3) + \omega^2 K_3(T_1^2 + T_2^2) +$$
$$\omega^2(T_1^2 + T_2^2 + T_3^2) - \omega^4 K_2K_3T_1^2T_2T_3 + \omega^4 K_3T_1^2T_2^2 + \omega^4(T_1^2T_2^2 + T_1^2T_3^2 + T_2^2T_3^2) + \omega^6 T_1^2T_2^2T_3^2. \tag{20}$$

In order to assure $|\varphi(\omega)| \leq \pi/2$ for any frequencies it is sufficient to have $d(\omega) \geq 0$ and $b(\omega) \geq 0$. This requirement is satisfied if we choose

$$T_1 \geq K_1T_2, \quad T_2 \geq K_2T_3. \tag{21}$$

Analogously, we obtain the following conditions for a four-stage amplifier:

$$T_1 \geq K_1T_2, \; T_2 \geq K_2T_3, \; T_3 \geq K_3T_{out}. \tag{22}$$

We shall determine the relationships between T_1, T_2, T_3 and T_{out} in the case where a definite attenuation γ of the logarithmic amplitude frequency response is specified for the amplifier with an open feedback loop.

Based on (3) and (10), we find

$$T_1 = \frac{1}{\omega_h} = \frac{T_{out}}{K_{out}} 10^{\frac{20 \log |K_{tr}(0)|}{\gamma}} \tag{23}$$

On the other hand, the time constants must be chosen to be proportional to their gains. For the case $\gamma =$ $= 20$ db/decade this is evident from relationships (22). Therefore, we may write

$$T_1 \approx \frac{K_1}{a} T_2, \qquad T_2 \approx \frac{K_2}{a} T_3, \qquad T_3 \approx \frac{K_3}{a} T_{out}, \tag{24}$$

where $\underline{a}$ is the unknown coefficient which must be determined.

From (24) it follows that

$$T_1 \approx \frac{K_1K_2K_3}{a^3} T_{out}. \tag{25}$$

Eliminating T_1 from (23) and (25), we find

$$a \approx \sqrt[3]{K_1K_2K_3K_{out}} \cdot 10^{\frac{-20 \log |K_{tr}(0)|}{3\gamma}} \tag{26}$$

Substituting the value which we have obtained for $\underline{a}$ into (24), we find the sought for relationship between the time constants which assure the attenuation γ db/decade:

$$T_1 \approx \frac{K_1T_2}{\sqrt[3]{K_1K_2K_3K_{out}}} 10^{\frac{20 \log |K_{tr}(0)|}{3\gamma}},$$
$$T_2 \approx \frac{K_2T_3}{\sqrt[3]{K_1K_2K_3K_{out}}} 10^{\frac{20 \log |K_{tr}(0)|}{3\gamma}}, \tag{27}$$
$$T_3 \approx \frac{K_3T_{out}}{\sqrt[3]{K_1K_2K_3K_{out}}} 10^{\frac{20 \log |K_{tr}(0)|}{3\gamma}}.$$

SUMMARY

There is proposed a principle of designing operational amplifiers which is based on cascade successive switching off. This principle permits to enlarge the pass band on approximately two orders. Brief description of operational amplifiers of two types designed according to the proposed principle and the data of their test which confirm theoretical results are given.

LITERATURE CITED

1. P. S. T. Buckerfield, The parallel – T.D.C. Amplifier. IEE, No. 71, vol 99, Oct. 1952.
2. B. Ya. Kogan, A. A. Maslov, and D. E. Polonnikov, "Electronic simulating equipment of the type "EMU-8A", Vestnik Akad. Nauk SSSR, 7 (1958).
3. C. S. Deering, A Wide-Band Direct-Coupled Operational Amplifier. Proceedings National Simulation Conference, January 19, 20, 21, 1956.
4. G. W. Smith and R. G. Wood. Principles of Analog Computation. McGraw-Hill Book Company, Inc. 1959.
5. B. Ya. Kogan, Electronic Simulators and Their Application in the Investigation of Automatic Control Systems [in Russian] (Fizmatgiz, 1959).
6. E. A. Goldberg, Stabilization of Wide-Band Direct Current Amplifier for Zero and Gain, RCA Review, T. 11, No. 2 (1950).
7. G. W. Bode, Circuit Theory and the Design of Amplifiers with Feedback [Russian translation] (Foreign Literature Press, 1948).

THE EFFECT OF THE CHARACTERISTICS OF AN ELECTRICAL ELEMENT ON THE CHOICE OF THE PARAMETERS FOR A HYDRAULIC AMPLIFIER

I. M. Krassov, L. I. Radovskii, and B. G. Turbin
(Moscow)
Translated from Avtomatika i Telemekhanika, Vol. 21, No. 12,
pp. 1623-1626, December, 1960
Original article submitted February 24, 1960

The paper provides a basic analysis of the computation of the initial pressure in the chamber between restrictors in a hydraulic amplifier of the jet-damper type which is controlled by an electromagnetic element; the characteristics of the latter are taken into account.

Usually the basic parameters of hydraulic amplifiers designed for operation in some automatic control systems are chosen without considering the properties and special features of the elements which control the amplifiers.

In this paper we study the effect of the characteristics of an electromagnetic control element of the type "REP" [1, 2] on the choice of the initial pressure in the chamber between restrictors in a jet-damper hydraulic amplifier.

For a jet-damper amplifier (Fig. 1) the dependence of the pressure p_1 in the inter-restrictor chamber on the angle of rotation φ of the damper in a steady-state mode of operation is given by the formula

$$p_1 = \frac{\sigma^2 p_0}{\sigma^2 + a^2 (h_0 + r\varphi)^2}, \tag{1}$$

where p_0 is the supply pressure for the amplifier, h_0 is the initial gap between the nozzle and the damper, $\underline{r}$ is the radius through which the damper can be "rocked", σ is the conductivity of the constant restrictor and is equal to

$$\sigma = \mu f \sqrt{\frac{2g}{\gamma}}, \tag{2}$$

$\underline{a}$ is a quantity which characterizes the conductivity of the nozzle-damper unit and is equal to

$$a = \mu_n \pi d_n \sqrt{\frac{2g}{\gamma}}. \tag{3}$$

In expressions (2) and (3) the quantities μ and μ_n respectively denote the discharge coefficients for the restrictor and the nozzle-damper unit, f is the flow area for the restrictor orifice, d_n is the diameter of the nozzle, γ is the specific weight of the working liquid, g is the acceleration of gravity. The direction in which the angle φ is measured is shown in Fig. 1.

Formula (1) is written for the case where the gap between the nozzle and the damper is increased for small angles of rotation of the damper. The quantity a is variable, since the coefficient depends on the gap h and the Reynolds number. For a small variation of the gap this quantity can be assumed approximately constant [3] and equal to the initial gap h_0; this assumption is made here.

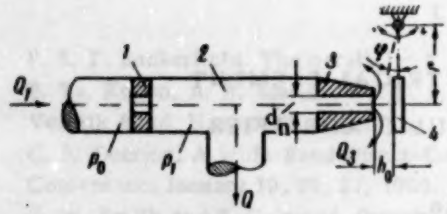

Fig. 1. Diagram illustrating the principle of a hydraulic jet-damper amplifier: 1) Nozzle with a constant flow cross section; 2) inter-restrictor chamber; 3) nozzle; 4) damper.

The achievement of high sensitivity and speed of response for the system requires that in the nominal mode of operation the amplifier must have its maximum pressure gain:

$$\left(\frac{\partial p_1}{\partial \varphi}\right)_{\varphi=0} = -\frac{2r p_{10}}{\lambda p_0}\sqrt{p_{10}(p_0 - p_{10})}, \tag{4}$$

where p_{10} is the initial steady-state value of the pressure in the chamber between restrictors,

$$p_{10} = \frac{\sigma^2 p_0}{\sigma^2 + a^2 h_0^2}, \tag{5}$$

$$\lambda = \frac{\sigma}{a}. \tag{6}$$

An analysis of the statics of just one hydraulic amplifier [4] shows that the maximum value of the pressure gain occurs for

$$p_{10} = 0.75\, p_0. \tag{7}$$

The jet of working liquid which emerges from the nozzle interacts with the damper and produces a moment M_h of the hydrodynamic forces. This moment represents the load on the control element and can be found either from tables [5] or by computation [6].

The static characteristic for the "REP" control element usually is of the form shown in Fig. 2, where $i_1, i_2, \ldots$, are the control currents; here $i_1 < i_2 < i_3 \ldots$.

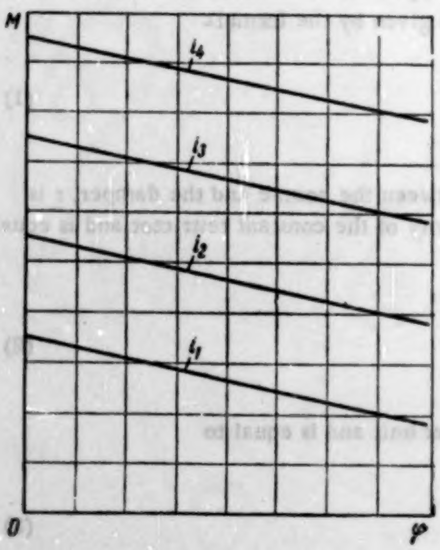

Fig. 2. Static characteristic for the electromagnetic control element.

The steady-state position of the damper is characterized by the equality of the electrical and hydrodynamic force moments:

$$M = M_h, \tag{8}$$

where M is the moment which moves the control element.

Taking into account the shape of static characteristics, we write

$$\frac{\partial M}{\partial i}\Delta i - \left|\frac{\partial M}{\partial \varphi}\right|\Delta\varphi = \frac{\partial M_h}{\partial \varphi}\Delta\varphi. \tag{9}$$

From this we obtain

$$\Delta\varphi = \frac{\frac{\partial M}{\partial i}}{\frac{\partial (M_h + |M|)}{\partial \varphi}}\Delta i. \tag{10}$$

Making use of (4) and (10) and taking into account the relationship

$$\left(\frac{\partial p_1}{\partial i}\right)_{\varphi=0} = \left(\frac{\partial p_1}{\partial \varphi}\right)_{\varphi=0}\left(\frac{\partial \varphi}{\partial i}\right)_{\varphi=0}, \tag{11}$$

we can write the gain of the amplifier for a nominal mode of operation for which $\varphi = 0$; we obtain an expression in the form of a current gain

$$\left(\frac{\partial p_1}{\partial i}\right)_{\varphi=0} = -\frac{2rp_{10}}{\lambda p_0}\sqrt{p_{10}(p_0 - p_{10})}\,\frac{\left(\frac{\partial M}{\partial i}\right)_{\varphi=0}}{\left|\frac{\partial M}{\partial \varphi}\right|_{\varphi=0} + \left(\frac{\partial M_r}{\partial \varphi}\right)_{\varphi=0}}. \tag{12}$$

For simplicity in notation we introduce the substitution

$$m_0 = \left(\frac{\partial M}{\partial i}\right)_{\varphi=0},\quad n_0 = \left|\frac{\partial M}{\partial \varphi}\right|_{\varphi=0},\quad q_0 = \left(\frac{\partial M_h}{\partial \varphi}\right)_{\varphi=0}, \tag{13}$$

where

$$m_0 = \text{const},\quad n_0 = \text{const}. \tag{14}$$

Based on experimental data, it was established that it is possible to assume

$$q_0 = c\bar{p}_{10}, \tag{15}$$

with a sufficient degree of accuracy; here $\underline{c}$ is a constant quantity, and $\bar{p}_{10}$ is the relative pressure in the inter-restrictor chamber which is equal to

$$\bar{p}_{10} = \frac{p_{10}}{p_0}. \tag{16}$$

In addition, we introduce the substitution

$$\alpha = \frac{n_0}{c}. \tag{17}$$

Then (12) can be rewritten in the form

$$\left(\frac{\partial p_1}{\partial i}\right)_{\varphi=0} = -\frac{2rp_0}{\lambda}\bar{p}_{10}\sqrt{\bar{p}_{10}(1-\bar{p}_{10})}\,\frac{m_0}{(\alpha+\bar{p}_{10})\,c}. \tag{18}$$

We determine the condition governing the existence of a maximum for the pressure gain. We formulate the equation

$$\frac{d}{d\bar{p}_{10}}\left(\frac{\partial p_1}{\partial i}\right)_{\varphi=0} = -\frac{2rp_0m_0}{\lambda(\alpha+\bar{p}_{10})\,c}\left[\frac{(1-2\bar{p}_{10})\,\bar{p}_{10}}{2\sqrt{\bar{p}_{10}(1-\bar{p}_{10})}} + \sqrt{\bar{p}_{10}(1-\bar{p}_{10})}\right] + \frac{2rp_0m_0}{\lambda(\alpha+\bar{p}_{10})^2c}\left[\sqrt{\bar{p}_{10}(1-\bar{p}_{10})}\,\bar{p}_{10}\right] = 0. \tag{19}$$

After certain simplifications, Eq. (19) becomes

$$\frac{rp_0m_0}{\lambda c}\,\frac{\bar{p}_{10}\left[2\bar{p}_{10}^2 + (4\alpha-1)\,\bar{p}_{10} - 3\alpha\right]}{(\alpha+\bar{p}_{10})^2\sqrt{\bar{p}_{10}(1-\bar{p}_{10})}} = 0. \tag{20}$$

Since the quantities r, $\bar{p}_0$, m_0, λ, c, α, p_{10} are positive, it follows that a solution of the equation is possible if

$$\bar{p}_{10}^2 + (2\alpha - 0.5)\,\bar{p}_{10} - 1{,}5\,\alpha = 0. \tag{21}$$

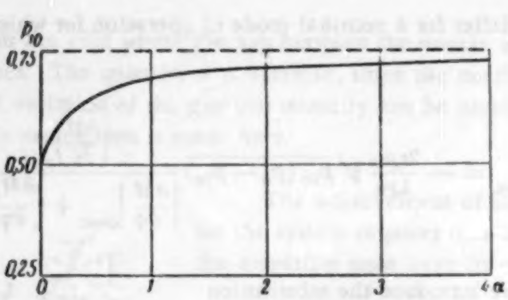

Fig. 3. Dependence of the relative pressure P_{10} on the quantity α.

From this we obtain

$$\bar{p}_{10} = \sqrt{\alpha^2 + \alpha + 0.0625} - (\alpha - 0.25). \tag{22}$$

if we take into account the fact that $\bar{p}_{10} > 0$.

Figure 3 shows the dependence of the relative pressure p_{10} on the quantity α. Examination of the curve shows that the relative pressure in the inter-restrictor chamber reaches the value 0.75 only for large values of α. For existing electromagnetic control elements and hydraulic amplifiers of the jet-damper type the value of α varies over the limits from 0.2 to 0.75; this corresponds to the initial segment of the curve $p_{10} = f(\alpha)$. For this reason it is necessary in a number of cases to take into account the effect of the control element on the operation of the hydraulic amplifier. Expression (22) makes it possible to determine the pressure p_{10} which assures the maximum pressure (or flow) gain as a function of the characteristics of the electrical control element and the characteristic of the nozzle-damper unit.

SUMMARY

There is well-grounded calculation of the initial pressure in the chamber between throttles of the hydraulic amplifier of the nozzle-flapper type which is controlled by an electromagnetic element, the characteristics of the element being taken into consideration.

LITERATURE CITED

1. Yu. M. Pul'er, "A contactless electromagnetic actuating and logometer mechanism," Priborostroenie, No. 1 (1957).
2. I. E. Dekabrun, N. P. Kozlov, and I. M. Krassov, "The dynamics of an electromagnetic control element," Avtomatika i Telemekhanika, 21, 4 (1960).*
3. I. M. Krassov and B. G. Turbin, Discharge Coefficients of a Hydraulic Jet-Damper Restrictor, Automatization of Production Processes, No. 3 (Academy of Sciences, USSR Press, 1960).
4. A. G. Shashkov, "Theory of a jet-damper control unit operating with oil," Avtomatika i Telemekhanika 17, 11 (1956).*
5. I. M Krassov and B. G. Turbin, "The force effect of a jet in a hydraulic jet-damper amplifier," Avtomatika i Telemekhanika 20, 12 (1959).*
6. I. M. Krassov, L. I. Radovskii, and B. G. Turbin, "Approximate determination of the reaction of the jet in a hydraulic amplifier," Avtomatika i Telemekhanika 21, 10 (1960).*

*See English translation.

LEV SEMENOVICH GOL'DFARB (ORBITUARY)

Translated from Avtomatika i Telemekhanika, Vol. 21, No. 12, pp. 1627-1628, December, 1960

On October 12, 1960, an outstanding scientist and leading specialist in automation, Doctor of Sciences Professor Lev Semenovich, Gol'dfarb, died.

Soviet science has suffered a heavy loss. We have lost a magnificent scientist, talented engineer, and a splendid teacher.

Upon completing the Moscow Power Institute, L. S. Gol'dfarb, worked since 1931 in the All-Union V. I. Lenin Electrical Engineering Institute (AEI). Soon he became one of the chief participants and directors of the staff that subsequently produced a number of great specialists.

In these years Lev Semenovich developed control systems for turbogenerators, control systems for governing the operation of boilers, automatic control systems for industry, etc. Simultaneously, he was performing important theoretical work on the investigation of stability and transient responses in linearized and nonlinear systems. It was in the AEI in 1940 that he originated ideas which led him to developing a method for studying self-oscillations which has now become classical. This method has been developed and applied in books and papers on the theory and design of automatic systems; it has justifiably been named the Gol'dfarb method.

In 1941, Lev Semenovich was called to serve in the ranks of the Soviet armed services; he served in the air force until the end of the Great Patriotic War. From 1945, until his death Lev Semenovich performed major and fruitful works in the Moscow Power Institute, first as a candidate and then (after defending his doctorate dissertation in 1948) as a professor on the automation staff.

The results of the basic papers by L. S. Gol'dfarb have been published in our journal, in which he took an active part.

Lev Semenovich worked without sparing time or energy. He gave generously of his experience to the younger generation and painstakingly educated specialists in the field of automation. He achieved success in this work due to his magnificent qualtities — a love of people, a personal warmth, a lively temperament, and a clear mind.

His memory will always be carried in the hearts of all those who had the priviledge of studying under him or of working with him.

A LIST OF LITERATURE ON MATHEMATICAL SIMULATION (ANALOG COMPUTERS) FOR 1958

(Translated from: Avtomatika i Telemekhanika Vol. 21, No. 12, 1629-1652, December, 1960.

CONTENTS

A. BOOKS

IN RUSSIAN

Belova A. I., Vitenberg I. M. and Gluzberg E. A. – A block for recording and reproducing functions (BRR) in the solution of integral equations. – Branch of the VINITI, AN SSSR, Moscow, 1958. 50 pp. with illustrations.

Bykhovskii M. L. – Fundamentals of the dynamic accuracy of electrical and mechanical networks. – AN SSSR, Moscow, 1958. 160 pp., 160 illustrations. Bibliography 89 titles. (Machinery Institute.)

Lebedev A. N. – Computer-decision units. – Mashgiz, Moscow, 1958. 378 pp., 216 illustrations. Bibliography 12 titles.

Problems in the synthesis and accuracy of complex analog devices (predominantly as they apply to mechanical computer-decision units). – AN SSSR, Moscow, 1958. 226 pp. with illustrations. (Machinery Institute.)

Problems in the theory of mathematical machines. Symposium 1. – Fizmatgiz, Moscow, 1958. 230 pp. with illustrations and tables.

IN LANGUAGES OTHER THAN RUSSIAN

Danlous-Dumesnils M. – Le calcul analogique pai

courant continu. – Paris, Dunod, 1958, 272 pp.
Karplus W. – Analog simulation. Solution of field problems. – New York, McGraw Hill, 1958, 449 pp., 241 illustrations, 21 tables.
Olson H. F. – Dynamic analogies. Ed. 2. – New York Van Nostrand, 1958, 278 pp.
Susskind A. K. ed. – Notes on analog digital conversion techniques. 2nd ed. – New York, Wiley and Sons, Inc., London, Chapman and Hall, 1958, 410 pp., ill.
Tomovic R. – Calculateurs analogiques repetifs. – Paris, Masson, 1958, 186 pp.

B. PROCEEDINGS OF CONVENTIONS AND CONFERENCES. INFORMATION

IN LANGUAGES OTHER THAN RUSSIAN

AIEE - IRE - ACM Eastern Joint Computer Conference. – Philadelphia, December 3-5, 1958. – Electr. Engng., 1958, Jan., vol. 77, No. 1, p. 73.
Analog-Rechenzentrum. – Elektro-Anz., 1958, No. 40-41. p. 383.
Association internationale pour le calcul analogique. – Rev. gen. mec., 1958, jan., an 42, No. 107, p. 50.
Central Simulation Council meeting of May 5 on non-linear phenomena. – Instrum. and Automat., 1958, vol. 31, No. 8, pp. 1388-1390.
Computer and Automation Conference, June 2-4, 1958, University of Texas, Austin, Tex. – Instrum. and Automat., 1958, May, vol. 31, No. 5, p. 881.
Eastern Simulation Council meeting of February, 1958 at Avco. – Instrum. and Automat., 1958, vol. 31, No. 8, pp. 1385-1386.
The Electronic Computer Exhibition. National Hall, Olympia, London, Nov. 28-Dec. 4, 1958. – Process Control and Automat., 1958, Oct., vol. 5, No. 10, p. 31.
Hatcher Ch. – Nel's new computer centre. Bur. Ships J., 1958, vol. 7, No. 6, pp. 14-16.
Joint meeting of January 8 on "Interrelationship of analog and digital computers." – Instrum. and Automat., 1958, vol. 31, No. 3, pp. 487-491.
Midwestern S/C meeting of June 9 on "Cute tricks in simulation." Instrum. and Automat., 1958, vol. 31, No. 10, Pt. 1, pp. 1691-1694.
National Simulation Conference (Dallas, Texas, Oct. 23-25, 1958). – Trans. IRE, 1958, March, vol. EC-7, No. 1, p. 75.
S/C meeting of Nov. 14 on "Standard simulation circuits." – Instrum. and Automat., 1958, Jan., vol. 31, No. 1, pp. 120-122.
SE S/C meeting of September 27 on analog-digital conversion. – Instrum. and Automat., 1958, Jan., vol. 31, No. 1, pp. 119-120.
Seconde Journées Internationales de Calcul Analogique. – Strasbourg, Sept. 1, 1958. – Mesures et controle industr., 1958, fevr., an. 23, No. 249, p. 130.
Symposium on computers. (Held at the Australian Administrative Staff College MT. Eliza, Victoria, December 3-6, 1957.) – Austral. J. Instrum. Technology, 1958, Febr., vol. 14, No. 1, pp. 5-18.
Jurgen R. K. – Technical highlights of '58 Wescon. – Electronics, 1958, Nov., vol. 31, No. 45, pp. 72-75, 8 ill.
Mussard J. A. – Un centre international provisoire de calcul à Rome. – Inform. Unescu, 1958, No. 284, pp. 7-9.
WS/C and DDA Council meeting of May 5 on "Problems unique to computers of a kind." Instrum. and Automat., 1958, vol. 31, No. 7, pp. 1219-1225.

C. GENERAL THEORETICAL PROBLEMS

1. General Problems

IN RUSSIAN

Bruevich N. G. – On the problem of the inputs and outputs of complex analog units. In: Problems in the synthesis and accuracy of complex analog devices. – AN SSSR, Moscow, 1958. pp. 7-36, 29 illustrations.
Fel'dbaum. – An automatic optimizer. – Avtomatika i Telemekhanika, 19, No. 8, pp. 731-743. 9 illustrations Bibliography 6 titles (1958).
Gluzberg E. A. – On the logic possibilities of mathematical machines of the analog type. In: Problems in the theory of mathematical machines, Symposium 1. – Fizmatgiz, Moscow, 1958, pp. 135-148, 14 illustrations. Bibliography 10 titles.
Khernauer V. – Computer devices. In: Engineering techniques for automization of production processes. – Mashgiz, Moscow, 1958, pp. 112-129, 12 illustrations.
Klishov N. A. The synthesis of computer-decision units which compute implicit functions by the method of approximations. In: Problems in the synthesis and accuracy of complex analog devices. – AN SSSR, Moscow, 1958, pp. 37-51, 4 illustrations. Bibliography 5 titles.
Kuz'minok G. K. Electrical integrators and the possibilities for their application in engineering-technological computations. In: Mechanization of design and computation. – Mashgiz, Moscow-Leningrad, pp. 27-53, 5 illustrations.
Semenov V. V. Electrodynamic simulation of the hydraulic section of a hydroelectric power station. – Nauchnye Doklady Vysshei Shkoly, Energetika, No. 2, 219-230 (1958).
Shuvalov N. K. Investigating transient responses

using electronic simulators. — Sudostroenie, No. 11, 41-45, 7 illustrations. Bibliography 6 titles. (1958).

Stakhovskii, R. I. A two-channel automatic optimizer. — Avtomatika i Telemekhanika, 19, No. 8, pp. 744-756, 18 illustrations (1958). •

LANGUAGES OTHER THAN RUSSIAN

AED computer linkage. — Res. and Engng., 1958, vol. 4, No. 1, p. 27.

Analogue computation. — J. Instn. Electr. Engrs., 1958, April, vol. 4, No. 40, p. 194-197, 5 illustrations.

Bauer W. F. — Aspects of real-time simulation. — Trans. IRE, 1958, vol. EC-7, No. 2, p. 134-150.

Blackman P. F. — A computer combining analog and digital principles for complex frequency network calculations. In: Proc. Conference on Computers in Control Systems, 1957. — New York, Am. Inst. Electr. Engrs., 1958, p. 34-41, 13 illustrations. Bibliography 88 titles.

Ernst D. Die Entwicklung der Rechenmaschinen in America. — Regelungstechnik, 1958, Jg. 6, H. 3, SS. 89-92, 5 illustrations. Bibliography 30 titles.

Ferguson A. E. Analogue computers. — Austral. J. Instrum. Technol., 1958, Febr., vol. 14, No. 1, p. 19-28, 10 illustrations. Bibliography 8.

Franks R. G. E. — Maximizing control performance and economy with analog simulation. — Am. J. Instrum. Soc., 1958, vol. 5, No. 9, pp. 80-94, 9 illustrations. Bibliography 5.

Gabillard R. — Les analyseurs differentiels electroniques. — Inds. atom, 1958, vol. 2, No. 7-8, pp. 91-107.

Gait J. J. — Analogue computers and their application. — Process Control and Automat., 1958, Nov., vol. 5, No. 11, pp. 480-485, 8 illustrations Bibliography 11.

Grosch H. R. J. — The future of computing. — Ind. Engng. Chem., 1958, Nov., vol. 50, No. 11, pp. 1664-1666.

Hare R. E. and Willison W. E. — Analogue computers. — Instrum. Practice. 1958, vol. 12, No. 11, pp. 1200-1207, 13 illustrations. No. 12, pp. 1295-1303, 18 illustrations.

Henle R. A and Walsh J. L. — The application of transistors to computers. — Proc. IRE, 1958, June, Vol. 46, No. 6, pp. 1240-1254, 52 ill. Bibliography 6.

Hirst F. — History of computers. Austral. J. Instrum. Technol., 1958, Feb., vol. 14, No. 1, pp. 14-18.

Hitcheon I. C. Simulation of nonlinear field problems. — Brit. Communs. and Electronics, 1958, Feb., vol. 5, No. 2, pp. 96-99, 5 illustrations. Bibliography 7.

Kleimpeter M. —Application d'un analyseur differentiel couplé à un microreseau pour l'étude du comportement des auxiliaires d'un grand alternateur lors d'un court circuit à ses bornes. — Rev. Gen. Electr., 1958, Dec., vol. 67, No. 12, pp. 685-691.

Leondes C. T. and Nothman M. — Real-time hybrid computers for control systems. In: Proc. Conference on Computers in Control Systems, 1957. New York, Am. Inst. Electr. Engrs., 1958, pp. 88-94, 10 illustrations. Bibliography 13.

Lonergan J. P. — An introduction to the application of analogue computers. Austral. J. Instrum. Technol., 1958, vol. 14, No. 1, pp. 62-71.

Mertens H. — Prinzip und Anwendung der Analog-Rechenmaschine. — Elektronik, 1958, Mai, jg. 7, Nr. 5, SS. 130-134, 11 illustrations.

New analog-digital-analog device. — Automat. Control, 1958, vol. 8, No. 6, p. 91.

Nicolau E., Simon N., Voiculescu Ch. and Weiss A. — Elemente neliniare pentru simulatore si calculatore analogice. Realizazi experimentale. — Automat. si electron., 1958, vol. 2, No. 2, pp. 74-77.

Nothman M. H. — Combined analog-digital control system. — Electr. Manufact., 1958, vol. 61, No. 6, pp. 78-86.

Palevsky M. — Calculateurs hybrides analogiques. — Automatisme, 1958, Mars, an 3, t. 3, No. 3, pp. 115-116, 8 illustrations.

Paul R. J. A. — Analogue simulation in system design. — Control, 1958, vol. 1, Nov., No. 5, pp. 221-226, 8 illustrations; Dec., No. 6, p. 276-278, 12 illustrations. Bibliography 15.

Pieretti J. So you have a problem. A fundamental discussion of analog and digital computers and comparison of their relative advantages and disadvantages. — Manhattan Coll. Engr., 1958, vol. 15, No. 3, pp. 94, 120-121, 4 illustrations. Bibliography 4.

Rao P. V. and Krishna G. — The transformer analog computer. — Commun and Electronics, 1958, Jan., No. 34, pp. 732-738, 18 illustrations. Bibliography 14.

Rogers S. — Analog computers and computing. — Ind. Eng. Chem., 1958, Nov., vol. 50, No. 11, p. 1636-1640, 4 illustrations.

Simulation speeds research at Bell Laboratories. — Bell Labs Rec., 1958, Sept., vol. 36, No. 9, pp. 348-349, 1 ill.

Sykes R. P. A preventive maintenance program for large general purpose electronic analog computers. — IRE National Convention Rec., 1958, vol. 6, Pt. 4, pp. 191-205, append., 13 illustrations.

Szamer B. — Ueber die Entwicklung wissenschaftlicher

Rechenmaschinen. – Neue tech. Buero, 1958, Bd. 2, Nr. 10, SS. 232-234, 3 illustrations.

Thermal computers. – Mech. Engng., 1958, vol. 80, No. 5, p. 86.

Thomas G. E. – Analogue computers as design tools. – Automat. Progr., 1958, vol. 3, No. 11, pp. 415-417, 429.

Unger H. – Grundlagen elektronischer Rechenanlagen. – Werkstattstechnik und Maschinenbau, 1958, Bd. 48, Nr. 2, SS. 125A-128A, 3 illustrations.

Venning B. H. – Analogue computers as an aid to teaching. – Bull. Electr. Engng. Educ., 1958, No. 21, pp. 41-54.

West G. P. – Computer control experience gained from operation of a large combined analog digital computation system. – In: Proc. Conference on Computers in Control Systems, 1957. – New York, Am. Inst. Electr. Engrs., 1958, pp. 95-97, 4 illustrations.

West, G. P. – The role of computers in analysis and design of control systems. – Trans. IRE, 1958, vol. AC, No. 5, p. 65.

Winkler H and Gumme G. – Die Bedeutung elektronischer Analogieanlagen für den Fortschritt der Elektrotechnik. – Dtsch. Elektrotechn., 1958, Sept., Bd. 12, Nr. 9, SS. 317-324, 23 illustrations. Bibliography 10.

2. Methods for Solving Problems Using Simulators

IN RUSSIAN

Lebedev A. N. – The necessary stability conditions for mathematical simulators in solving systems of algebraic and transcendental equations with three or more unknowns by the method of choosing roots. – Izv. Vysshikh Uchebnykh Zavedenii, Priborostroenie, 1958, No. 6, pp. 38-49, 5 illustrations.

Sirazitdinov E. G. and Petrova E A. – Simulating actuating mechanisms in automatic control systems. – Izv. Vysshikh Uchebnykh Zavedenii. Priborostroenie, 1958, No. 3, pp. 63-72.

Smolov V. B. – On a certain method for simulating integro-differential functions. – Izv. Vysshikh Uchebnykh Zavedenii. Priborostroenie, 1958, No. 4, pp. 69-75, 5 illustrations.

LANGUAGES OTHER THAN RUSSIAN

Bauer W. F. – Aspects of real-time simulation. – Trans. IRE, 1958, June, vol. EC 7, No. 2, pp. 134-136.

Buehler E. – Die elektronische Nachbildung mechanischer Reibung – Regelungstechnik, 1958, Jg. 6, H. 1, SS. 14-19, 14 illustrations.

Ernst D. – Das Arbeiten mit Analogie Rechengeraeten. – Regelungstechnik, 1958, Jg. 6, H. 6, SS. 206-209, 3 illustrations. Bibliography 16.

Greenspan D – Note on analog techniques for solving two-point boundary value problems. – Rev. Sci. Instrum., 1958, Sept., vol. 29, No. 9, pp. 787 788, 1 illustration.

Herrmann H. – Aids to programming techniques for electronic analogue computers. – Abhandl. Braunschw. Wiss. Gesell., 1958, vol. 10, pp. 117-149.

Herschel R. and Kettel E. – Das Problem der Normierung bei der Anwendung von Analog-Rechenmaschinen zur Behandlung regelungstechnischer Aufgaben. – Regelungstechnik, 1958, Jg. 6, H. 11, SS. 400-405, 6 illustrations. Bibliography 6.

Horn R. E. and Honnell P. M. – Matrix programming of electronic analog computers. – Commun. and Electronics, 1958, Sept., No. 38, pp. 420-426, 11 illustrations.

Lane L. J. – A method of scaling and checking computer circuits. – Applic. and Ind., 1958, May, No. 36, pp. 67-70, 3 illustrations.

Matyas J. Vicenasobne elektricke integratory. – Slaboproudy Obzor, 1958, t. 19, No. 11, str. 758-765.

Napodobeni prubehu mechanickeho treni servomechanismo na elektronickem modelu. – Automatisace, 1958, No. 10, str. 344-345.

Nathan A. – Computing and error matrices in linear differential analyzers. – Trans. IRE, 1958, March, vol. EC-7, No. 1, pp. 32-33, ill. Bibliography.

Otterman J. – on the loop- and node-analysis approaches to the simulation of electrical networks. – Trans. IRE, 1958, Sept., vol. EC-7, No. 3 3, pp. 199-206, 10 illustrations.

Scheinman A. H. – A numerical-graphical method for synthesizing switching circuits. – Commun. and Electronics, 1958, Jan., No. 34, pp. 687-689, 7 illustrations.

Storey L. R. O. and Grierson J. K. Time-symmetric filters. I. Methods of simulation. – Electronic Engng., 1958, Oct., vol. 30, No. 368, p. 586-592, 5 illustrations. Bibliography 7.

Tomovic R. – Method of iteration and analog computation. – Ann. Assoc. Internat. Calcul. Analog., 1958, vol. 1, No. 2, pp. 60-63.

3. The Accuracy of Simulating Units and Their Elements

IN RUSSIAN

Ordanovich A.E. – The effect of nonideal characteris in integrators and adders on the operation of electronic simulators. – Nauchnye Doklady vysshei shkoly,

Fiziko-Matematicheskie Nauk. 1,155–161,1 illustrations (1958).

Problems in the synthesis and accuracy of complex analog devices (AN SSSR, Moscow, 1958. 226 pages with illustrations; Machinery Institute)

Smolov V. B. and Smirnov A. N. – Computing the accuracy of an electromechanical integrator and certain possible methods for increasing the accuracy. – Izvestiya Leningradskovo Elektrotekhnicheskovo Instituta, 1958, No. 34, pp. 34-45 3 illustrations.

Tetel'baum I. M. and Chelnokov N. N. – Increasing the accuracy with which problems can be solved on an electronic analog computer. – Transactions of the Moscow Power Institute, 27 (Gosenergoizdat, Moscow-Leningrad, 1958, pp. 259-266.)

LANGUAGES OTHER THAN RUSSIAN

Dow P. C. – An analysis of certain errors in electronic differential analyzers. Capacitor dielectric absorption. – Trans. IRE, 1958, March, vol. EC-7, No. 1, pp. 6-16, 14 illustrations. Bibliography 8.

Fixa Z. – Stanoveni presnosti analogového pocitace v cecelem oboru jeho cinnosti (Determining the accuracy of a specialized analog computer) – Stroje zpracov infirm., 1958, No. 6, pp. 153-184.

Kley A. – Einflüsse des Nullpunktfehlers und des Gitterstromes von Gleichstromverstärkern bei der Lösung linearer Differentialgleichungen mit konstanten Koeffizienten auf dem elektronischen Analogrechner. – Elektron. Rundschau, 1958, Sept., Bd. 12, No. 9, SS. 305, 8 illustrations. Bibliography 6.

Miura T. and Nagata M. – Theoretical consideration of computing errors of a slow type electronic analog computer. – Trans. IRE, 1958, Dec., vol. EC-7, No. 4, pp. 306-312, 8 illustrations, 6 tables.

Noda K. and Kurokawa K. "Estimating the error in an electrical simulator" [in Japanese with an English Abstract], Bull. Electrotechn. Lab., vol. 22, No. 3, 189-203 and 233 (1958).

Overhoff M. W. – Etude de certaines erreurs dans les calculateurs analogiques. – Ann. telecommuns, 1958, juil-août, 1958, t. 13, No. 7-8, pp. 162-168, 8 illustrations. Bibliography 6.

Petras S. – Presnost niesenia uloh na elektronickych analogovych pocitacich strojoch. (The accuracy of solving problems on electronic analog computers), Strojnoelektrotechn. casop., 1958, t. 9, No. 10, str. 602-613.

D. SIMULATING DEVICES OF THE INDIRECT ANALOGY TYPE

I. Electronic Devices

IN RUSSIAN

Kogan B. Ya., Maslov A. A. and Polonnikov D. E. – Electronic simulating equipment of the type "EMU-8A". – Vestnik Akad. Nauk, 1958, No. 7, pp. 69-74, 5 illustrations.

Petrov G. M. and Stepin V. S. – A small electronic simulator "MNM". (Central Scientific-Research Institute of Complex Automatization, Instruments and means of automatization). – Moscow, 1958. 8 pp. with illustrations.

Sitnikov O. P. and Chapunov R. P. – A dc electronic simulator. – Vestnik Vyisshei Shkoly, 1958, No. 7, p. 83.

Vitenberg I. M. – Problems of designing a high speed electrical simulator for analyzing dynamic systems. – In: Problems in the theory of mathematical machines. – Fizmatgiz, Moscow, 1958, pp. 149-166, 11 illustrations. Bibliography 14.

Vitenberg I. M. – An electronic simulator for the automatic selection of the optimal solutions of the problem for a specified system of equations. – In: Automatic control and computer engineering. – Mashgiz, Moscow, 1958. pp. 419-437, 17 illustrations.

LANGUAGES OTHER THAN RUSSIAN

Analogue computer. – Automat. Progr., 1958, vol. 3, No. 10, p. 392.

Analogue computer. – Electronic Engng, 1958, Oct., vol. 30, No. 368, p. 620.

Analogue computer. – Instrum. Rev., 1958, vol. 5, No. 59, p. 1083.

Analogue computer components. – Process Control and Automat., 1958, Jan., vol. 5, No. 1, pp. 28-29.

Analogue equipment. – Instrum. Rev., 1958, vol. 5, No. 61, p. 1169.

Analog-Rechner Type A-938. – Automatik, 1958, Bd. 3, Nr 10, S. 271.

Bevc V. – Primena elektrichih analognih sklopova u resavanju diferencijalnih jednacina. (The use of electrical simulating networks for solving differential equations) – Elektrotehn vesn., 1958, vol. 12, No. 9-10, pp. 292-295 (Yugoslavia).

Building block expansion in new analogue computer. – Automat. Control, 1958, vol. 8, No. 1, p. 45.

Calculateur analogique à 12 amplificateurs. – Mesures et contrôle industr., 1958, vol. 23, No. 256, pp. 711-713.

Camatini C., Gupido A. – La calcolatrice analogica elettronica nello studio dei probleme di regolazione automatica. – Dell Inform. CGE, 1958, vol. 19, No. 1, pp. 2-15.

Choundhury A. K. and Nag B. R. – An electronic differential analyser. – Indian J. Phys., 1958, Febr.,

vol. 32, No. 2, pp. 91-108, illustrations.
Computer compresses real time by 1/3000.— Electr. Design News, 1958, vol. 3, No. 10, p. 3.
De Vos J.— Aufbau und Anwendung elektronischer Analogo-Rechner.— Werkstattstechnik und Maschinenbau, 1958, Febr., Jg. 48, H. 2, SS. 131A-132A, 4 illustrations.
Dhen W.— Aufbau und Anwendung elektronischer Analogrechner.— VDI-Berichte, 1958, Bd. 30, SS. 27-34, 10 illustrations. Bibliography 7.
Elektronikus analog szamologepek. (Electronic analog computers) — Radiotechnika, 1958, vol. 8, No. 12, pp. 357-358 (Hungary).
EMIAC-II.— E. M. I.'s analog computer.— Brit. Communs and Electronics, 1958, June, vol. 5, No. 6, pp. 444-445.
Ernst D.— Aufbau und Wirkungsweise elektronischer Analogiegerate.— Regelungstechnik, 1958, Jg. 6, H. 4, SS. 125-128, 13 illustrations; H. 5, SS. 162-167, 7 illustrations. Bibliography 6.
Ferguson A. E.— Analogue computers.— Austral. J., Instrum. Technol., 1958, Febr., vol. 14, No. 1, pp. 19-28, 10 illustrations. Bibliography 8.
Findeisen W.— Maszyny matematyczne analogowe na Wystawie "INTERKAMA 1957". (Analog Computers at the "Interkam" exhibition in 1957) — Pomiary, Automatika, Kontrola, 1958, No. 3, str. 124, 2 illustrations.
Gabillard R.— Les analyseurs differentiels electroniques.— Ind. atom., 1958, vol. 2, No. 11-12, pp. 44-58.
Gandusio G. and Guerrini F.— Calcolatrice analogica elementare per il controllo della produzione.— Strum. e automaz., 1958, vol. 6, No. 3, pp. 108-111.
Hare R. E. and Willison W. E.— Analogue computers.— Instrum. Practice, 1958, vol. 12, No. 12, pp. 1295-1303.
Henning T.— Elektroniska analoregnemaskiners oppbygging og anvendelse.— Tekn. ukebl., 1958, vol. 105, No. 2, pp. 21-28.
Jiewertz B.— Saabs elektroniska differential-analysator seda.— Tekn. tidskr., 1958, vol. 88, No. 31, pp. 773-780.
Macacu Apau.— Description of an analog computer.— J. Instrum. Control Assoc. Japan, 1958, vol. 7, No. 6, pp. 4-10.
The "Minispace" analogue computer.— Process Control and Automat., 1958, Dec., vol. 5, No. 10, pp. 417, 1 illustration.
Miura T. and Namakura T.— General description of an analog device. Hitachi Hyoron, 1958, No. 27, pp. 51-60.
Nag B. R.— An electronic differential analyser.— Indian J. Phys., 1958, Febr., vol. 32, pp. 91-108.
New analog computer speeds patching.— Control Engng., 1958, vol. 5, No. 2, p. 138.
New computer can be "tailored" to meet all needs.— Office Equipm. Ind., 1958, No. 6.
Oever M. J., ven den.— Symposium over elektronische analogon machines. De analogen rekenmaschine van de sub-afdeling der vliegtugbouwkunde der technische Hogeschool te Delft.,— Ingenieur (Nederl.), 1958, vol. 70, No. 10, pp. 029-033.
Saro analogue computer at de Havillands.— Brit. Communs. and Electronics, 1958, Feb., vol. 5, No 2, p. 116, I illustration.
Siniti Ikesawa.— An electronic simulator.— Sangyo Kikai, No. 89, pp. 25-31.
Stubbs G. S.— SADSAC: A sampled-data simulator and computer using stepping relays.— Automat. Control, 1958, Apr., vol. 8, No. 4, pp. 47-50, 6 illustrations.
Thomas E. L.— Analogue computation.— Brit. Communs and Electronics, 1958, May, vol. 5, No. 5, pp. 348-358, 14 illustrations.
Tomovic R. and Mitrovic D.— Some experiences with a repetitive differential analyser.— Bull. Inst. Nucl. Sci., 1958, vol. 8, No. 144, pp. 109-116.
U.S.A. Beckman EASE 1100 analogue computer.— Interavia Air Letter, 1958, No. 4071, p. 5.
Wechselstrom-Analogie-Rechengerät Typ IA 55.— Schweiz. Techn., 1958, Nr 2, S. 45.
Winkler H.— Methoden und Probleme der Rechenelektronik in Analogieverfahren— Technik, 1958, Jan, Bd. 13, Nr. I, SS. 6-10, 11 illustrations.
Zusatzgerät zur Nachbildung einer Zeitkonstanten für das Analogie-Rechengerät AR. 2.— Regelungstechnik, 1958, Bd. 6, Nr 9, SS. 348-349.

II. Decision Elements in Electronic Devices

1. DC Electronic Amplifiers

IN RUSSIAN

Polonnikov D. E.— The input circuits of amplifiers with contact choppers.— Avtomatika i Telemekhanika, 1958, vol. 19, No. 6, pp. 582-591, 4 illustrations. Bibliography 6.
Smolov V. B.— A decision amplifier with a differential input.— Avtomatika i Telemekhanika, vol. 19, No. 12, pp. 1145-1149.
Takhvanov G. I.— Feedback in decision amplifiers.— Transactions of the Ryazan' Radio-Engineering Institute, 1958, vol. 3, pp. 128-138, 8 illustrations.

LANGUAGES OTHER THAN RUSSIAN

Bekey G. A.— Basic components of analog computers.— 1958, Proc. Instrum. Soc. Amer. 13-th Annual

Instrum.— Automat. Confer. and Exhibit. Pt. 2.— Pittsburgh, Instrum. Soc. Amer., 1958, Part PTC 3, pp. 1-14, 16 illustrations. Bibliography 24.

Chan Go-In.— Designing dc amplifiers for electronic simulators.— Itsi Yui Shian Uzishu, 1958, No. 3, pp. 40-43 (Chinese People's Republic), no. 1, p. 198.

Chopper stabilized PC amplifiers in new desktop analogue computer.— Automat. Control, 1958, June, vol. 8, No. 6, p. 90.

Depian L. and Smith R. E.— A stabilized D-C differential transistor amplifier.— Commun. and Electronics, 1958, May, No. 36, pp. 157-159, 10 illustrations. Bibliography 8.

Hutcheon J. C.— Performance calculations for D. C. chopper amplifiers.— Electronic Engng, 1958, Aug., vol. 30, No. 366, pp. 476-480, 7 illustrations. Bibliography 7.

Kabes K. and Tikal F.— Cs. soucastky pro analogove matematicke stroje. (Czeckoslovak parts for analog computers).— Slaboproudý Obzor, 1958, sv.19, No. 1, str. 13-19, 16 illustrations. Bibliography 12.

Klein P. E.— Doppel-Differenzier-Verstärker.—Elektronik, 1958, Bd. 7, Nr. 4, SS. 100-102, 6 illustrations.

Martin D. J.— New types of D. C. amplifier. Pt. 1. The cascade balance system.— Electronic and Radio Engr, 1958, Jan., vol. 35, new ser., No. 1, pp. 2-7, illustrations. Bibliography 7.

Maynard F. B. and Steele E. L.— A high frequency P-N-P switching transistor.— Proc. Nat. Electronics Conference 1957, vol. 13.—Chicago, Illinois, Nat. Electronics Confer., Inc., 1958, pp. 246-253, 8 illustrations. Bibliography 6.

Nitzberg R.— An improved operational amplifier.— Proc IRE, 1958, March, vol. 46, No. 3, pp. 614-615, 1 illustration.

Rao J. V.— Electronic analogue computers, 1. The adding circuit.— J. Instn. Telecommun. Engrs, 1958, vol. 4, No. 3, pp. 143-146.

Williams A. J., Eynon J. U. and Polster N. E.— Some advances in transistor modulators for precise measurement.— Proc. Nat. Electronics Conference 1957, vol. 13.— Chicago, Illinois, Nat. Electronics Conf., Inc., 1958, pp. 40-54, 10 illustrations. Bibliography 8.

2. Decision Amplifiers Without Vacuum Tubes

IN RUSSIAN

Dobrov E. V.— The effect of temperature instability of a germanium transistor modulator on the operation of a dc operational amplifier.— Priborostroenie, 1958, No. 3, pp. 12-15, 5 illustrations.

Milovzorov V. P.— Magnetic summing amplifiers.— Transactions of the Ryazan Radio-Engineering Institute, vol. 3.— Soviet Radio, 1958, pp. 95-103, 6 illustrations.

Petrov G. M. and Popov V. A.— On designing an operational amplifier using semiconductor elements.— Priborostroenie, 1958, No. 3, pp. 4-8, 12 illustrations.

Sapozhkov K. A.— Semiconductor instruments in summing decision units.— Transactions of the Ryazan Radio-Engineering Institute, vol. 3.— Soviet Radio, pp. 115-123, 5 illustrations.

LANGUAGES OTHER THAN RUSSIAN

Blecher F. H.— Transistor multiple loop feedback amplifiers.— Proc. Nat. Electronics Conf., 1957, vol. 13.— Chicago, Illinois, Nat. Electronics Conf., Inc., 1958, pp. 19-34, 10 illustrations. Bibliography 14.

Giorgis J. and Thompson C. C.— Silicon transistor performance in a chopper application.— Applic. and Ind., 1958, July, No. 37, p. 113-119, 12 illustrations. Bibliography 8.

Krause C. A. and Lowe R. R.— Design of AC computing amplifiers using transistors.— Trans. IRE, 1958, Sept., vol. EC-7, No. 3, pp. 191-195, 8 illustrations.

Schenkerman S.— Designing transistor D-C to A-C converters.— Electronics, Engng issue, 1958, Sept., vol. 31, No. 39, pp. 78-80, 3 illustrations.

3. Integrating and Differentiating Devices

Anna O.— Elektrische Integrationsverfahren.— Elektron. Rundschau, 1958, Jg. 12, Nr. 4, SS. 127-128.

Clegg J. C.— A nonlinear integrator for servomechanisms.— Applic. and Ind., 1958, March, No. 35, pp. 41-42, 5 illustrations.

Hisey B. L. and Perl E. R.— Electronic integrator with immediate digital output.— Rev. Scient. Instrum., 1958, May, vol. 29, No. 5, pp. 335-359, 6 illustrations. Bibliography 10.

Hodara H.— H-F wide-band electronic integrator design.— Electronic Industr. 1958, Oct., vol. 17, No. 10, pp. 96-100.

Integrator.— Instrum. and Automat., 1958, Nov., vol. 31, No. 11, pp. 66-68, 12 illustrations.

Johnson W.— A differentiator for a-c. computers.— Commun. and Electronics, 1958, March, No. 35, pp. 1-4, 6 illustrations. Bibliography 14.

Ranga-Rao J. V.— Electronic analogue computers. II. The integrator.— J. Instn Telecommun. Engrs, 1958, vol. 5, No. 1, pp. 33-37.

Sharp J. and Williams R. W.— An a.c. velodyne integrator.— J. Scient. Instrum., 1958, April vol. 35, No. 4, pp. 125-129, illustrations.

4. Multiplying-Dividing Devices

IN RUSSIAN

Burgin B. Sh.— Electrical multiplication and division and its application in instrumentation and control.— Elektrichestvo, 1958, No. 9, pp. 60-62, 4 illustrations.

Ismailov Sh. Yu. and Smolyarov A. M.— Load characteristics of multiplying units containing quadratic elements.— Transactions of the Ryazan' Radio-Engineering Institute, vol. 3.— Soviet Radio, 1953, pp. 34-54, 5 illustrations.

Oranskii A. M.— The application of a pulsed multiplying unit.— Transactions of the Ryazan' Radio-Engineering Institute, vol. 3.— Soviet Radio, 1958, pp. 104-114, 10 illustrations.

Steinov Gr., Derzhinskii Al. and Dmitriev M.— Electronic devices for simulating products and for dividing power functions.— Doklady Bolgarskoi Akademy Nauk, 1958, vol. 11, No. 6, pp. 465-468.

Vaisman L. V.— An electrical device for multiplication and division.— Priborostronie, 1958, No. 8, p. 16, 2 illustrations.

LANGUAGES OTHER THAN RUSSIAN

Analog multiplier.— Rev. Scient. Instrum., 1958, vol. 29, No. 12, p. 1166.

Chasmar R. P. and Cohen E.— An electrical multiplier utilizing the Hall effect in indium arsenide.— Electronic Engng., 1958, Nov., vol. 30, No 369, pp. 661-664, 5 illustrations. Bibliography 10.

Choudhury A. K. and Nag B. R.— Analogue multiplier and function generator with cathode-ray tube.— Ind. J. Phys., 1958, March, vol. 32, pp. 141-148.

Garcia Santesmaseo J. and Civot Bren A.— Multiplicador y divisor analogico a integraciones periodicas.— An. Real soc. esp. fis. y quim., 1958, A54 No. 5-6, pp. 127-138.

Hilsum C.— Multiplication by semiconductors.— Electronic Engng. 1958, Nov., vol. 30, No. 369, pp. 664-666, 7 illustrations. Bibliography 6.

Lax Ch. and Rafoeld Ph. I.— Mathematical elements for analog computers.— Instrum. and Automat., 1958, vol. 31, No. 11, pp. 1832-1833.

Laefgren L.— Analog multiplier based on the Hall effect.— J. Appl. Phys., 1958, Feb., vol. 29, No. 2, pp. 158-166, 7 illustrations. Bibliography 12.

Multiplier unit for analogue computers.— J. Scient. Instrum., 1958, vol. 35, No. 10, pp. 389.

Oxenius J.— Die Verwendung von Hall Generatoren in Analog Multiplikatoren.— Nachrichtentechn. Z., 1958, Mai, Bd. 11, Nr 5, SS. 263-268.

Riss J.— Analogova nasobicka vyuzivajici Hallova efektu. (A multiplying device which uses the Hall effect).— Sdelovaci techn., 1958, t. 6, No. 9, str. 346.

Schmid H.— A transistorized four-quadrant time-division multiplier with an accuracy of 0.1%.— Trans. IRE, 1958, vol. EC-7, No. 1, pp. 41-47.

Sideriades L. and Brunel J.— Multiplicateur de tensions pour calculatrices electroniques.— Electronique industr., 1958, No. 22, pp. 161-164.

Zeilinger K.— Wirkungsweise und Aufbau von elektromechanischen Analog-Multiplikatoren.— Elektron. Rundschau, 1958, Sept., Bd. 12, Nr. 9, SS. 307-308, 4 illustrations.

5. Function Generators

IN RUSSIAN

Ismailov Sh. Yu. and Smolyarov A. M.— The accuracy of reproducing monatomic functions using rectifier function generators.— Transactions of the Ryazan' Radio-Engineering Institute, vol. 3.— Soviet Radio, 1958, pp. 44-53, 5 illustrations. Bibliography 5.

Lebedev A. N.— Approximate simulation of certain nonlinear functions of two independent variables.— Transactions of the Ryazan' Radio-Engineering Institute, vol. 3.— Soviet Radio, 1958, pp. 74-77.

Maslov A. A.— Diode function generators of a specialized type based on semiconductor elements.— In: Automation and Remote Control.— AN SSSR, Moscow, 1958, pp. 41-48.

Maslov A. A. and Talantsev A. D.— Cathode-ray function generators based on the principle of a controlled sweep.— In: Automation and Remote Control.— AN SSSR, Moscow, 1958, pp. 49-57.

Pul'er Yu. M.— Contactless sine-cosine rotary transformer-selsyns.— Elektrichestvo, 1958, No. 1, pp. 5-9, 5 illustrations. Bibliography 6.

Rastopshin A. S.— The application of semiconductors in computer-decision instruments for the purpose of designing functional four-terminal networks.— In: The application of semiconductors in instrumentation.— Mashgiz, Moscow, 1958, pp. 83-90.

Smolov V. B.— A universal diode function generator.— Izvestiya Vysshikh Uchebnykh Zavednii, Priborostroenie, 1958, No. 5, pp. 14-18, 3 illustrations.

LANGUAGES OTHER THAN RUSSIAN

Bekey G. A.— The function generator as a basic tool for nonlinear control systems.— Automat. Control, 1958, Aug., vol. 9, No. 2, pp. 42-46, 13 illustrations.

Brown E.— The design of function generators using silicon carbide nonlinear resistors.— Electronic Engng, 1958, March, vol. 30, No. 361, pp. 154-157, illustrations.

Card programmed function generators.— Automat. Control, 1958, April, vol. 8, No. 4, p. 52.

Derzhansky A. and Stainov G.— Electronic circuit modelling a power function.— Doklady Bolgarskoi Akademy Nauk, 1958, vol. 11, No. 4, pp. 271-274, 4 illustrations.

Freudenstein F.— Four-bar function generators.— Trans. Fifth Conference on Mechanisms. (Purdue Univ. Oct. 13-14, 1958).— Cleveland, Penton Publ. Co., 1958, pp. 104-107, I illustration. Bibliography 3.

Function generation for process control simulation.— Proc. Instrum. Soc. Amer. 13th Annual Instrum.-Automat. Confer. and Exhibit, 1958, Pt. 2.— Pittsburgh, Instrum. Soc. Amer., 1958, pp. 1-23, 7 illustrations. Bibliography 24.

Function generator for two independent variables.— Instrum. Practice, 1958, July, vol. 12, No. 7, p. 769, 2 illustrations; Techn. News Bull. Nat. Bur. Standards, 1958, Apr., vol. 42, No. 4, pp. 67-70, illustrations.

Function generator uses optics.— Control Engng, 1958, vol. 5, No. 2, p. 139.

Garcia Santesmases J. and Gonzales Ibeas J.— Un Generador de functiones de diodes con modulation de señal.— An Real soc. esp. fis. y quim., 1958, vol. A54, No. 9-10, pp. 263-280.

Hellstrom M. J.— Hyperbolic analogs.— Proc. IRE, 1958, Febr., vol. 46, No. 2, p. 502, 2 illustrations.

Holbrook G. W.— Hyperbolic analogs using varistors.— Proc. IRE, 1958, vol. 46, No. 10, p. 1762.

Hybrid loop generators square root functions.— Electr. Design News, 1958, vol. 3, No. 3, pp. 22-23.

Inada Nobukazu.— A function generator.— Keisoku, J. Soc. Instrum. Technol. Japan, 1958, vol. 8, No. 10, p. 622-626.

Kelley T. J.— Multi-variable function generator.— Instrum. and Automat., 1958, Nov., vol. 31, No. 11, p. 1819, 2 illustrations.

Kovach L. D. and Comley W.— Nonlinear transfer functions with thyrite.— Trans. IRE, 1958, June, vol. EC-7, No. 2, pp. 91-97, 6 illustrations.

Lax Ch. and Rafield Ph. I.— Mathematical elements for analog computers.— Instrum. and Automat., 1958, vol. 31, No. 11, pp. 1832-1833, 5 illustrations.

Liesegang H.— Ein logaritmischer Gleichstromverstärker mit selbsttätiger Driftkompensation und kurzer Einstellzeit.— Arch. techn. Messen, 1958, Nr 268, SS. R68-R71.

Nicolau E., Simion N., Voiculescu Ch. and Weiss A.— Elemente neliniare pentru simulatore si calculatore analogica. Realizazi Esperimentale.— Automat. si electron., 1958, vol. 2, No. 2, pp. 74-77.

Revay A. W. and Ford D. J.— Design of function generators. Electric Engng., 1958, Oct., vol. 77, No. 10, pp. 927-928, 2 illustrations.

Revay A. W. and Ford D. J.— The design of function generators using short-time memory devices and nonlinear elements.— Communs. and Electronics, 1958, May, No. 36, pp. 143-152, 18 illustrations.

Schmid H.— A transistorized all-electronic cosine/sine function generator.— IRE Wescon Convent Rec., 1958, vol. 2, Pt. 4, pp. 89-107.

Scott L.— Adjustable nonlinear function generator.— Electronics, Engng ed., 1958, vol. 31, No. 27, pp. 84, 86-88.

Silverberg B.— Application techniques for arbitrary function generation.— Automat. Control, 1958, vol. 9, No. 4, pp. 17-19; No. 5, pp. 6-9, 4 illustrations.

Susini A.— Un generatore di funzione a diodi.— Elettrotechnica, 1958, Nov., vol. 45, No. 11, pp. 626-629, 11 illustrations.

Tomovich R.— New applications of an electronic function generator.— Trans. IRE, 1958, March, vol. EC-7, No. 1, pp. 48-51, 10 illustrations.

Williams R. W.— Resistance potentiometers as function generators.— Electronic Engng, 1958, Oct., vol. 30, No. 368, pp. 579-585, 23 illustrations. Bibliography 6.

Willis J.— A cathode-tube function generator.— Electronic Engng., 1958, vol. 30, No. 465, p. 458.

6. Other Decision Elements and Auxiliary Equipment

LANGUAGES OTHER THAN RUSSIAN

Douce J. L., Graduate Ph. D. and West J. C.— A magnetic-drum store for analogue computing.— Proc. Inst. Electr. Engrs, 1958, Nov., vol. 105, Pt. B., No. 24, pp. 577-580, 4 illustrations. Bibliography 11.

Gibson W. E.— Three axis flight simulator systems.— Aero-Space Engng., 1958, Aug., vol. 17, No. 8, pp. 36-39, 5 illustrations.

Hellwarth G. A.— A constant-amplitude random function generator.— Communs. and Electronics, 1958, Sept., No. 38, pp. 443-452, 15 illustrations.

Holzman E. G.— Variable transport lag simulation.— Preprint Nucl. Engng. and Sci Confer. s.a. No. 112, 27 p., illustrations.

Kozak W. S.— New job for an old method: capacitor storage used in analogue memory.— Canad. Electronics Engng., 1958, Feb., vol. 2, No. 2, pp. 38-42.

Lebrun J.— Les potentiomètres bobinés speciaux utilisés en calcul analogique.— Electronique, 1958, août 15, No. 143, pp. 33-38, 21 illustrations.

McLeod J.— Survey of recorders being used with analog computers.— Instrum. and Automat., 1958, May, vol. 31, No. 5, p. 852.

Mitchel J.— An accurate analog frequency measuring circuit.— Electr. Engng., 1958, Oct., vol. 77,

No. 10, pp. 912-914, 6 illustrations.

Nestorescu D. N.— Despre stabilitatea surselor di alimentare pentru mazina analoaga de calcui.— Studiisi cercetari energ. Acad RPR, 1958, vol. 8, No. 2, pp. 304-308.

III. Electromechanical Devices

IN RUSSIAN

Baranchuk E.I.— On the theory of electromechanical differential analyzers.— Izvestiya Vysshikh Uchebnykh Zavedenii, Priborostroenie, 1958, No. 4, pp. 76-86, 4 illustrations. Bibliography 11.

Baranchuk, E. I.— On the stability and dynamic accuracy of electromechanical computer-decision units.— Izvestiya Vysshikh Uchebnykh Zavedenii, Priborostroenie, 1958, No. 2, pp. 52-62, 7 illustrations. Bibliography 7.

Gorkii V. G.— A rotary-amplifier computer.— Avtomatika i Telemekhanika, 1958, vol. 19, No. 5, pp. 448-455, 8 illustrations. Bibliography 5.

Ikhntykan A. B. and Kharkevich Yu. F.— This was achieved in a university. New means for the mechanization of mathematical computation.— Vestnik Vysshei Shkoly 1, 64-65 (1958).

LANGUAGES OTHER THAN RUSSIAN

Bell J.— An analogue computer, employing magslips and serves.— Technique, 1958, vol. 12, No. 3, pp. 20-23.

A mechanical analogue computer.— Brit. Communs. and Electronics, 1958, June, vol. 5, No. 6, p. 445, 1 illustration.

Michales G. W.— Mechanical analog computer components.— Trans. Fifth Conference on Mechanisms.— Cleveland, Penton Publ. Co., 1958, pp. 108-139, 40 illustrations. Bibliography 9.

Rogers Ch. E. and Ake J. E.— Unilog— a continuous rate computer.— ISA Journal, 1958, vol. 5, No. 2, p. 63.

Sager G.— Zum Bau der dritten deutschen Gezeitenrechenmaschine.— Feingerätetechnik, 1958, Bd. 7, Nr 12, SS. 555-559.

Zeilinger K.— Beitrag zum Aufbau einfacher Analog-Rechengerate mit elektromechanischen Bauelementen fur die Mess- und Regeltechnik.— Elektronik, 1958, Mai, Jg. 7, Nr 55, SS. 135-139, 10 illustrations. Bibliography 10.

IV. Pneumatic-Hydraulic Devices

IN RUSSIAN

Ivlichev Yu. I. and Nadzhafov E. M.— A universal pneumatic multiplier-divider unit and a unit for extracting the square root.— Avtomatika i Telemekhanika, 1958, vol. 19, No. 11, pp. 997-1009, 8 illustrations.

Kuz'manko L. V.— Slotted hydraulic integrators.— Nauchnye Doklady Vysshei Shkoly, Elektromekhanika i Avtomatika, 1958, No. 2, pp. 85-93, 3 illustrations.

V. Specialized Devices

1. Devices for Solving Systems of Algebraic Equations. Root Finders

IN RUSSIAN

Brick V. A. and Ginzburg S. A.— A computer which plots conformal representations for a power-law polynomial.— Avtomatika i Telemekhanika, 1958, vol. 19, No. 7, pp. 674-683, 6 illustrations. Bibliography 10.

Daube Ya. Ya.— A transformer analog for linear algebraic relationships.— Nauchnye Doklady Vysshei Shkoly, Energetika, 1958, No. 3, pp. 211-220, 6 illustrations. Bibliography 6.

Lebedev A. N.— On various methods for formulating a mathematical model for solving equations by the method of choosing roots.— Transactions of the Ryazan' Radio-Engineering Institute, vol. 3.— Soviet Radio, 1958, pp. 78-85, 6 illustrations.

Mikhailov N. N.— On plotting root loci for automatic control systems.— Avtomatika i Telemekhanika, 1958, vol. 19, No. 7, pp. 661-673, 9 illustrations. Bibliography 10.

Mikhailov N. N.— Electrical devices for solving algebraic equations (Survey).— Avtomatika i Telemekhanika, 1958, vol. 19, No. 5, pp. 477-490, 12 illustrations. Bibliography 24.

LANGUAGES OTHER THAN RUSSIAN

Elektrischer Analogrechner zur Ausgleichrechnung.— Vermessungstechn. Rundschau, 1958, Bd. 20, Nr 10, SS. 349-350, 1 illustration.

Hempel E.— Ueber eine elektronische Analogie— Rechenanlage zur Auflösung von linearen Gleichungssystemen.— Nachrichtentechnik, 1958, Okt., Jg. 8, H. 10, SS. 453-455, 5 illustrations.

Krishnamurthy E. V. and Ramachandran G. N.— "Lilavati"— a new analogue computer for solving linear simultaneous equations and related problems. Pt. 1.— Proc. Indian Acad. Sci., 1958, Sept., vol. 48, Sect. A., No. 3, pp. 152-164, 8 illustrations. Bibliography 12.

Krishnamurthy E. V.— "Lilavati"— a new analogue computer for solving linear simultaneous equations and related problems. Pt. II. Design of model II and its application to the solution of secular equations.— Proc. Indian Acad. Sci., 1958, vol. A48, No. 5, pp. 269-283.

Lukaszewicz L.— Analizator townan algebraicznych

linowych aral. (An analyzer for systems of linear algebraic equations).— Biul. zakl. aparat. mat. PAN 1958, P. I., N 5, pp. 0-19 (in Polish).

Mrkvicka J.— Elektrickenalogove stroje na reseni soustav linearnich algebraickych rovnic. (Electrical computers for solving systems of linear algebraic equations).— Stroje zpracov. inform., 1958, N 6, p. 185-208.

Nag B. R.— On the application of an electronic differential analyser for finding the roots of a polynomial.— Indian J. Phys. and Proc., 1958, May, vol. 32, No. 5, pp. 212-217, illustrations.

"Wina" network analyser.— Engineer, 1958, vol. 205, No. 5339, pp. 770-771.

2. Devices for Solving Integral Equations

IN RUSSIAN

Val'denberg Yu. S.— Computer method for solving a certain class of integral equations.— Avtomatika i Telemekhanika, 1958, vol. 19, No. 8, pp. 725-730, 2 illustrations. Bibliography 9.

Val'denberg Yu. S.— On the principles of designing an analog computer for solving integral equations. In: Automatic Control and Computer Engineering.— Mashgiz, Moscow, 1958, pp. 399-418, 6 illustrations. Bibliography 10.

Val'denberg Yu. S.— The use of a potentialoscope for realizing the iterative process in an analog computer.— Priborostroenie, 1958, No. 7, pp. 22-24, 2 illustrations. Bibliography 5.

Val'denberg Yu. S.— Development of the theory and designing a computer that solves integral equations.— Author's abstract of candidate's dissertation.— Moscow, 1958. 12 pp. (Moscow Higher Engineering School.)

Val'denberg Yu. S.— A specialized analog computer for solving integral equations.— In: Theses of the reports to the convention on computer mathematics and the application of the means of computer engineering.— Academy of Sciences, Azerbaidzhan SSR, Baku, 1958, pp. 57-58.

Vasil'ev V. G.— An automatic device for solving Volterra equations of the second kind with a kernel of the type K(t − x).— Nauchnye Doklady Vysshei Shkoly, Elektromekhnika i Avtomatika, 1958, No. 4, pp. 36-38.

LANGUAGES OTHER THAN RUSSIAN

Fisher M.— Proposed methods for the analog solution of Fredholm's integral equation.— J. Assoc. Comput. Machinery, 1958, Oct., vol. 5, No. 4, pp. 357-369. Bibliography 20.

Cabillard R.— Les analyseurs differentiels electroniques (Utilisation).— Ind. Atom., 1958, vol. 2, No. 11-12, pp. 41-58.

Zeleznikar A.— Rjesavanje nehomogenih sistema linearnih integralno-diferencijalnichjednadzhi viseg reda a konstantinim koeficijentima pomocu istornjemih elektroniskih analognih racunala. (Solutions of a system of inhomogeneous integral-differential equations of high order using dc electronic analog computers.)— Elektrotehn, vesn., 1958, vol. 12, N 5-6, pp. 177-187 (in Yugoslav, with German abstract.)

3. Correlators

IN RUSSIAN

Novikov Yu. V.— Specialized analog computer for the statistical manipulation of random processes.— In: Automatic Control and Computer Engineering.— Mashgiz, Moscow, 1958, pp. 375-398, 17 illustrations.

Samoilov A. I.— An instrument for instantaneous correlation conversion of functions.— In: Symposium of the Leningrad Institute of Railroad Transport Engineers, No. 158.— Transzheldorizdat, 1958, pp. 352-367, 6 illustrations. Bibliography 16.

LANGUAGES OTHER THAN RUSSIAN

Eddins W. T.— Sample-and-hold circuits for time correlation of analog-voltage information.— Convent. Rec. IRE, 1958, vol. 6, Rt. 5, pp. 21-24, 4 illustrations.

Shintaro Oshima, Kensuke Takashima, Katsumi Nishitor.— Reps. Electr. Commun. Labor. Nippon Telegr. and Teleph. Public Corp., 1958, May, vol. 6, No. 5, pp. 135-144, 12 illustrations.

4. Various Devices

LANGUAGES OTHER THAN RUSSIAN

Azaroff L. V.— Semiautomatic two-dimensional Fourier analog computer.— Rev. Scient. Instrum., 1958, vol. 29, No. 4, pp. 317-318.

Timbrell V.— A calculator for numerical Fourier synthesis.— J. Scient. Instrum., 1958, vol. 35, No. 9, pp. 313-318, 11 illustrations.

Tucker D. G.— An analogue computer Fourier transforms.— J. Brit. Instn Radio Engrs, 1958, Apr., vol. 18, No. 4, pp. 233-236, 2 illustrations. Bibliography 7.

Zaroodny S. I. and Leser T.— Aydar special purpose analog machine for yaev data reduction.— J. Assoc. Comput. Machinery, 1958, Jan., vol. 5, No. 1, pp. 89-99, illustrations.

Schmidt J. D., Putschi H. N. and Keonjian E.— Transistorized special purpose computer.— Proc. Nat. Electronics. Conference, vol. 13.— Chicago Nat.

Electronics Conference, Inc., 1958, pp. 624-634, 12 illustrations.

E. DEVICES FOR CONVERTING A DIGITAL CODE TO PHYSICAL QUANTITIES AND VICE VERSA

IN RUSSIAN

Anisomov B. V. and Vinogradov Yu. V.— Certain problems in the accuracy of devices which convert a voltage to a digital code with feedback.— Nauchnye Doklady Vysshei Shkoly, Mashinostroenie i Prioborostroenie, 1958, No. 4, pp. 210-219.

Baltrushevich A. V.— Systems for converting a digital code into an angle of rotation or into a linear displacement.— Nauchnye Doklady Vysshei Shkoly, Elektromekhnika i Avtomatika, 1958, No. 2, pp. 59-67, 12 illustrations. Bibliography 21 (1958)

Ryzhov V. I.— Devices for converting analog quantities into codes and codes into analog quantities.— In: Automatic control and computer engineering.— Mashgiz, Moscow, 1958, pp. 243-264, 27 illustrations. Bibliography 15.

Vinogradov Yu. V.— Analyzing the accuracy of operation for a high speed device which converts a voltage into a digital code.— Author's abstract of candidate's dissertation.— Moscow, 1958, 15 pages; (Moscow Higher Engineering School).

LANGUAGES OTHER THAN RUSSIAN

Abberigi A., Bernardini C. and Quercial F.— Proposal for an analog-to-digital electronic converter suited for nuclear pulse height analysis.— Nucl. Instrum., 1958, vol. 3, No. 4, pp. 201-203, 6 illustrations.

Bauwens J. C.— Study of a digital—analogue converter.— Rev. HF, Brussels, 1958, vol. 4, No. 2, pp. 39-42.

Couleur J. F.— BIDEC, a binary—to decimal or decimal—to binary converter.— Trans. IRE, 1958, Dec., vol. EC-7, No. 4, pp. 313-316, 6 illustrations.

DeBey L. G. and Webb R. C.— A shaft position digitizer system of high precision.— IRE Nat. Convent. Rec., 1958, vol. 6, No. 5, pp. 204-210.

Digital communications for analogue computers.— Process Control and Automat., 1958, Nov., vol.5, No. 11, pp. 498-499, 2 illustrations.

Double voltage comparison improves converter stability.— Electr. Design News, 1958, vol. 3, No. 8, pp. 14-15.

Electronic digitizer.— Automat. Progr., 1958, vol. 3, No. 5, p. 198.

Error-free analog-digital converter.— Electron Design, 1958, vol. 6, No. 14, pp. 32-33.

Fuwa Hiroshi, Kimura Iwao and Jitsukawa Taku.— An automatic calibrator for an analog-digital converter.— Bull. Electrotech. Lab., 1958, vol. 22, No. 8, pp. 583-596 and 632-633.

Fuwa Hiroshi.— Conversion of analog quantities to digital quantities. Solution performed in 1/200 seconds.— Automation, 1958, vol. 3, No. 1, pp. 44-48 (in Japanese).

Hofheimer R. W. and Perry K. E.— Digital-analog function generators.— Trans., 1958, June, vol. 1-7, No. 2, pp. 111-117, 16 illustrations, 1 table.

Ichikava — An analog-to-digital converter of the relay type.— J. Railway Engng. Res., 1958, vol. 15, No. 4, p. 180.

Ichikawa Kunihiko — A relay servosystem — an analog-to-digital converter.— Automat. Control., 1958, vol. 5, No. 4, pp. 112-120.

Kompass E. J.— What about digital transducers?— Control Engng., 1958, vol. 5, N 7, pp. 94-99.

McMillan S. H. and Sutton W. A.— A transistorized six-channel airborne digitizer.— IRE Convent. Rec., 1958, vol. 6, No. 5, pp. 26-33.

Miniaturized analog-to-digital shaft converter increases resolution, elliminates ambiguity.— Military Automat.; 1958, vol. 2, No. 5, pp. 174-175.

Miura Takeo.— An analog-to-digital converter.— J. Instrum. Soc. Japan, 1958, vol. 8, No. 11, pp. 748-749.

Nakagawa Takayuki.— An analog-to-digital converter using a transistor and an iron core.— J. Appl. Phys. Japan, 1958, vol. 27, No. 4, pp. 232-235.

New analog-digital and digital-analog device.— Automat. Control, 1958, vol. 8, No. 6, p. 91.

Nottebohm H.— Ein Analog-digital-Umsetzer für Strom, Spannung, Widerstand und Kapazität.— Elektron. Rundschau, 1958, März, Nr 3, SS. 85-88, 6 illustrations.

Palevsky M.— A new approach to analog digital conversion.— Instrum. Soc. Amer. Journal, 1958, April, vol. 5, No. 4, pp. 42-44, 6 illustrations.

Palevsky M.— A solid state analog-to-digital conversion device.— IRE Nat. Convent Rec., 1958, vol. 6, Pt. 4, pp. 232-235, 4 illustrations.

Photo-electro-mechanical digitalizer.— Electron. Ind., 1958, vol. 17, No. 4, pp. 149-150, 2 illustrations.

Rosmann F.— Les convertisseurs analogiques-numeriques.— Automation, 1958, vol. 4, No. 21, pp. 141-145.

Rowe W. D.— A transistorized digital-to-analog converter.— Trans. IRE, 1958, March, vol. 1-7, No. 1, pp. 22-28, 13 illustrations.

Sawaragi Voshikazu, Akashi Haijime and Okada Tsuyoshi.— The construction and testing of an analog-to-digital-converter.— Trans. Japan Soc. Mech. Engrs, 1958, vol.24, No.143, pp.517-522

Susskind A. K.— Characteristics and appraisal of analog-digital conversion techniques.— In: Proc. Computers in Control Systems Conference, 1957.— New York, Amer. Inst. Electr. Engrs, 1958, pp. 26-29, 5 illustrations.

Togino Kazuto.— A converter which converts a digital quantity into a physical quantity.— J. Instrum. Soc. Technol. Japan, 1958, vol. 11, pp. 758-759.

Towles W. B.— Transistorized analog-digital converter.— Electronics, 1958, Aug., 1, vol. 31, No. 31, pp. 90-93, 5 illustrations.

Walton C. A.— A direct-reading printed circuit commutator for analogue-to-digital data conversion.— IBM. J. Res. and Developm., 1958, July, vol. 2, No. 3, pp. 178-192.

Willis J. and Hartley M. G.— A cathode-ray tube analogue-to-serial digital converter.— J. Sci. Instrum., 1958, June, vol. 35, No. 6, pp. 197-202, 13 illustrations.

F. THE USE OF SIMULATING DEVICES OF THE INDIRECT ANALOGY TYPE

1. The Use of Simulators for Solving Problems in Automatic Control

IN RUSSIAN

Batkov A. M.— Analysis and synthesis of linear automatic control systems using simulators.— In: Automatic control and computer engineering.— Mashgiz, Moscow, 1958, pp. 438-463, 4 illustrations. Bibliography 16.

Belyaev I. P.— Mathematical simulation of dc rotary amplifier networks.— Izvestiya Vysshikh Uchebnykh Zavedenii, Elektromekhanika, 1958, No. 1, pp. 21-34, 14 illustrations. Bibliography 8.

Efroimovich Yu. E.— A computer for controlling the operation of an arc furnace.— Elektrichestvo, 1958, No. 5, pp. 15-19, 5 illustrations.

Efroimovich Yu. E.— Simulators for remote control of electrical quantities in arc furnaces.— Elektrichestvo, 1958, No. 3, pp. 22-27, 5 illustrations. Bibliography 5.

Monastyrshin G. I.— Mathematical simulation of dry friction.— Avtomatika i Telemekhanika, 1958, vol. 19, No. 12, pp. 1091-1106, 13 illustrations. Bibliography 8.

Sirazitdinov E. G. and Petrova E. A.— Simulation of actuators in automatic control systems.— Izvestiya Vysshikh Uchebnykh Zavedenii, Priborostroenie, 1958, No. 3, p. 6, 6 illustrations. Bibliography 7.

Solodov A. V.— Statistical investigation of nonstationary processes in linear systems using inverse simulators.— Avtomatika i Telemekhanika, 1958, vol. 19, No. 4, pp. 312-324, 8 illustrations.

Veiershtein R. A. and Fel'dbaum A. A.— Using an electronic simulator to design a system network which is close to optimal.— Avtomatika i Telemekhanika, 1958, vol. 19, No. 9, pp. 824-835, 14 illustrations. Bibliography 9.

LANGUAGES OTHER THAN RUSSIAN

Amber G. H. and Amber P. S.— Special purpose computers in the control of continuous processes.— Automatic Control, 1958, May, vol. 8, No. 5, pp. 43-48, illustrations.

Bigelow S. C.— The design of analog computer compensated control systems.— Applicat. and Ind., 1958, Nov., No. 39, pp. 409-415, 7 illustrations. Bibliography 11.

Black K. G. and Noorda R.— Analog computer study of wind tunnel drive.— Communs. and Electronics, 1958, Jan., No. 34, pp. 745-750, 9 illustrations. Bibliography 8.

Bradburn W. J.— Analysis of a motor speed-control system with an analog computer.— Trans. IRE, 1958, April, vol. PGIE-5, pp. 94-102, 11 illustrations.

Braun E. L. and Post G.— Systems considerations for computers in process control.— IRE Convent. Rec., 1958, vol. 6, Pt. 4, pp. 168-181, 6 illustrations.

Brown B. P.— Ground simulator studies of the effects of valve friction, stick friction, flexibility, and backlash on power control system quality-NACA, 1958, Rep. 1348, 13 p.

Chestnut H., Dabul A. and Leiby D.— Analog computer study of sampled-data systems.— In: Proc. Computers in Control Systems Conference, 1957.— New York, Amer. Inst. Electr. Engrs, 1958, pp. 71-76, 18 illustrations.

Clymer A. B.— Direct system synthesis by means of computers.— Communs. and Electronics, 1958, vol. 77, pp. 798-806.

Csordas Z.— Pneumatikus szabalyozas — modell: a korszeru folyamatszabalyozas segedeszoze.—(A pneumatic simulator — a modern means for investigating automatic control systems) — Meres es automat., 1958, vol. 6, N 2-3, pp. 84-89.

Elgerd O. I.— An analog computer study of the transient behavior and stability characteristics of serial-type digital data systems.— Communs. and Electronics, 1958, July, No. 37, pp. 358-366, 27 illustrations. Bibliography 15.

Field W. B.— Analog simulation for process design and control.— Proc. Instrum. Soc. Amer. 13th Annual Instrum. Automat. Conf. and Exhibit. Pt. 2.— Pittsburgh, Instrum. Soc. Amer., 1958, pp. 1-13, 8 illustrations, 2 tables. Bibliography 9.

Goggio E. C.— Some applications of analog-computer techniques to control system design.— Trans. IRE, April, vol. PGIE-5, pp. 70-76, 9 illustrations.

Holzman E. G.— Variable lag simulation. Preprint Nucl. Engng and Sci Conf. s.a. No. 112, 27 p., illustrations.

Petras St.— Elektronicke modelovania viacparametrovych regulacnych obvodov. (Electrical simulation of control systems with several parameters)— Strojnoelektrotechn. casop., 1958, t. 9, N 4, str. 217-230.

Prescott H. L.— Computer analyses voltage-regulator performance.— Westinghouse Engr, 1958, vol. 18, No. 3, pp. 76-79.

Prescott H. L.— Step regulator design checked on analog computer.— Electr. World, 1958, vol. 145, No. 18, pp. 56-57.

Schwartzenberg J. W.— An analog study of a high-speed recording servomechanisms.— Trans ASME, 1958, February, vol. 80, No. 2, pp. 490-496, 14 illustrations. Bibliography 6.

Straight P. E.— Effects of limiting error in feedback analysis.— J.Instrum. Soc. Amer., 1958, vol. 5, No. 5, pp. 44-49,, 11 illustrations.

Tallman G. H. and Smith O. J. M.— Analog study of dead-beat posicast control.— Trans. IRE, 1958, March, vol. PGAC-4, pp. 14-21, 10 illustrations. Bibliography 6.

Woods F. A.— Simulation of process control with an analog computer.— Industr. Engng Chem., 1958, November, vol. 50, No. 11, pp. 1627-1630, 9 illustrations.

II. The Use of Simulators and their Elements in Aviation

LANGUAGES OTHER THAN RUSSIAN

Computers and simulators.— Flight, 1958, vol. 74, No. 2590, pp. 460-461.

Controls fly trainer-almost.— Prod. Engng., 1958, July 14, vol. 29, No. 28, p. 71.

Creel H. F.— The weapons system approach to air data computing.— Aeronaut. Engng. Rev., 1958, February, vol. 17, No. 2, pp. 41-44, 50, 3 illustrations.

Dynamic simulatoring aids Short's VTOL research.— Brit. Communs and Electronics, 1958, August, vol. 5, No. 8, p. 618, 3 illustrations.

Heising C. R.— Evaluation of a turbojet control on an engine simulator.— In: Proc. Computers in Control Systems Conference, 1957.— New York, Amer. Inst. Electr. Engrs, 1958, pp. 54-57, 8 illustrations.

Hendrix G. E.— Target simulator tests beam-rider missiles.— Electronics, 1958, January 31, vol. 31, No. 5, pp. 32-35.

Hollander G. L.— Real-time airborne digital computers.— J. Instrum. Society of America, 1958, vol. 5, No. 12, pp. 26-32, 6 illustrations. Bibliography 6.

Mc Williams A. R.— An electronic brain.— Wings (S. Air U), 1958, February 17.

Nakada Y. and Scroggs S. J.— Simulation techniques in the synthesis of automatic flight-control systems.— In: Proc. Computers in Control Systems Conference, 1957.— New York, Amer. Inst. Electr. Engrs, 1958, pp. 57-59, 2 illustrations.

Schwamm E. G.— Electronic simulation for the jet age.— Aero/Space Engng., 1958, May, vol. 17, No. 5, pp. 34-37, 2 illustrations.

Simplin K. H.— Computers for simulators: A.C. or D. C.? — Flight, 1958, July 4, vol. 74, No. 2580, pp. 7-9, 5 illustrations. Bibliography 8.

III. The Use of Simulators in Nuclear Engineering

IN RUSSIAN

Chadwik G. L. and Thompson T.— Electrical simulation of the process which occurs in a reactor.— Khimiya i Khimicheskaya Tekhnologiya, 1958, No. 12, pp. 112-120, 2 illustrations.

Korolev V. V. and Sidorova L. I.— Electrical simulation of nuclear reactors.— Atomnaya Energiya, 1958, No. 1, pp. 29-43, 13 illustrations. Bibliography 27.

Leiby D. W.— The use of analog computers in designing an atomic electric power station. — Voprosy Yadernoi Energetiki, 1958, No. 4, pp. 59-72, 18 illustrations.

Toma V.— The Electronic digital computer in the institute of atomic physics of the Academy of the Roumanian People's Republic.— Byull. nauchn. inform., 1958, No. 1, pp. 121-123, (Academy of the Roumanian People's Republic, Roumanian-Soviet Scientific Institute).

LANGUAGES OTHER THAN RUSSIAN

Albrecht J.— Zur elektronischen Analogie ungedämpfter gekoppelter mechanischer Schwingungen.— Wiss. Z. Hochschule Elektrotechnik Ilmenau, 1958, Jg. 4, H I, SS. 33-39, 5 illustrations. Bibliography 11

Braffort P. and Caillet C.— Nouvelles techniques dans la simulation des centrales nucleaires.— Onde electr., 1956, août—sept., an. 38, No. 377-378, pp. 583-591, 13 illustrations. Bibliography 5.

Carteron J. M. and Deloux G.— Analog representation of heat exchange application to the simulation of heat exchangers of nuclear power plants.— In: Proc. computers in control systems conference, 1957.— New York, Amer. Inst. Electr. Engrs, 1958, pp. 48-53, 12 illustrations.

Computers in nuclear-reactor design.— J. Instn. Electr.

Engrs, 1958, May, vol. 4, No. 41, pp. 249-251.

Computer-run submarine simulator provides new dockside training.— Res. and Engng, 1958, vol. 4, No. 1, p. 43.

Electronic simulator for nuclear reactor power plants.— Engineer, 1958, December 5, vol. 206, No. 5367, p. 894.

Hanus B., Balgar J., and Mikulek M.— Elektricky analogon vymeniku tepla. (An electrical simulator for a heat-exchanger) — Slaboproudy Obzor 1958, sv. 19, No. 1, str. 36-41, 19 illustrations.

Johnson S. O., Curlee N. J. and Reihing J. V.— Simulation of hot channel boiling in water-cooled reactors.— Trans. IRE, 1958, June, vol. NS-5, No. 1, pp. 1-9, 9 illustrations.

Kourim G.— Ein elektromechanischer Reaktor —Simulatoren.— Regelungstechnik, 1958, Jg. 6, H. 5, SS. 185-188, 100 illustrations. Bibliography 9.

Kourim G.— Reaktor — Simulatoren.— Regelungstechnik, 1958, Jg. 6, H. 4, SS. 134-138, 8 illustrations. Bibliography 8.

Leiby D. W.— Analog computer study of the transient performance of a dual-cycle boiling-water-reactor nuclear power plant.— Communs and Electronics, 1958, March, No. 35, pp. 17-25, 18 illustrations. Bibliography 10.

Lynch E. E. and Waite D. P.— Steam-generator instrumentation for a nuclear power plant with analog computer verification of dynamic action.— Communs. and Electronics, 1958, November, No. 39, pp. 690-696.

Miroux J.— L'utilisation des signaux carres dans la technique des analogies electriques. Appareillage à grandes resistances pour curves rheoelectriques.— Onde electr. 1958, vol. 38, No. 375, pp. 450-457.

Nuclear reactor simulator.— Brit. Engng., 1958, vol. 40, No. 10, p. 334.

Nutter J. C.— Analogue computers in the nuclear power programme.— Instrum. Practice, 1958, January, vol. 12, No. 1, pp. 46-51, 4 illustrations. Bibliography 5.

Paul L. I. and Cox L. E. G.— Xenon-poisoning computer.— Nucleonics, 1958, vol. 16, No. 5, pp. 97-101.

Power reactor simulator.— Nuclear Engng., 1958, April, vol. 3, No. 25, p. 163.

A power reactor simulator — English electric's new model.— Nucl. Power 1958, vol. 3, No. 23, pp. 123-134.

Silks T. M. and Brondel J. O.— Analog simulation of a nuclear power plant.— Proc. Instrum. Soc. Amer. 13th Annual Instrum. Automat. Confer. and Exhibit, 1958. Pt. 2.— Pittsburgh, Instrum. Soc. Amer., 1958, Part PCT9, pp. 1-14, 12 illustrations. Bibliography 8.

Waite D. and Lynch E. E.— Computer verification of steam generator instrumentation for a nuclear power plant.— In: Proc. Computers in Control Systems Conference, 1957.— New York, Amer. Inst. Electr. Engrs, 1958, pp. 42-47, 14 illustrations.

Wilson I. and Potter R.— Analogue computers and their use in nuclear reactor safety studies.— J. Brit. Inst. Radio Engrs, 1958, February, vol. 18, No. 2, pp. 95-100, 5 illustrations.

Wilson I. and Lawrence L. A. J.— A nuclear power plant training simulator for use at Calder Hall.— J. Brit. Instn. Radio Engrs, 1958, February, vol. 18, No. 2, pp. 85-94, 9 illustrations.

Ziegler W.— Concerning a simple reactor analogue.— Nucleonik, 1958, April, vol. 1, No. 1, pp. 27-29.

IV. Various Applications

IN RUSSIAN

Gitel'man A. I.— Adjustment of the control equipment and the supply sources of gas-turbine motors using an electronic analog computer.— Energomashinostroenie, 1958, No. 10, pp. 30-34, 11 illustrations.

Golomidov I. P. and Lyubynskii E. A.— Mathematical simulation of a control system for an excavator electric drive with a compensated longitudinal-field amplifier.— Izvestiya Vysshikh Uchebnykh Zavedenii, Gornyi Zhurnal, 1958, No. 3, pp. 105-110, 3 illustrations.

Maslovskii P. M.— On the possibility of using computers in systems for the automatization of open-hearth furnaces.— Izvestiya Vysshikh Uchebnykh Zavednii, Chernye Metally, 1958, No. 5, pp. 77-88, 1 illustration.

Rotenberg R. V.— The use of electronic digital and analog computers for studying the vibrations in an automobile.— Avtomobil'naya Promyshlennost' 1958, No. 1, pp. 5-9.

Sitnik P. Kh.— Mathematical simulation of collectorless ac machines.— Izvestiya Vysshkh Uchebnykh Zavedenii, Elektromekhnika, 1958, No. 1, pp. 35-49, 8 illustrations.

Sitnik M. Kh.— Mathematical simulation of asymmetrical machines.— Izvestiya Vysshykh Uchebnykh Zavedenii, Elektromekhanika, 1958, No. 8, pp. 28-44, 14 illustrations. Bibliography 5.

Tarasov V. S., Oksenov B. E. and Butomo I. D.— The use of electronic computers for solving problems in various fields of engineering physics.— In: The Symposium: Radio physics.— Mashgiz, Moscow-Leningrad, 1958. pp. 22-240, 18 illustrations; (Leningrad Polytechnic Institute, Transactions No. 194).

Use of analog computers in shipbuilding.— Collection of abstracts of Symposium of References to foreign papers on shipbuilding, Sudpromgiz., 1958. No. 44, pp. 3-13, 11 illustrations. Bibliography 8.

Williams T., Johnson P. and Ross A.— The application of computer engineering in chemical technology.— Khimiya i Khimicheskaya Tekhnologiya, 1958, No. 7, pp. 212-233, 9 illustrations. Bibliography 53.

LANGUAGES OTHER THAN RUSSIAN

Adams J. W.— Reactor power calculator. Preprint. Nucl. Engng. Sci. Conf. s.a. No. 175, 10 p., 4 illustrations.

Analogue computer applied to inductionmotor design.— Engineering, 1958, August 8, vol. 186, No. 4822, p. 188, 2 illustrations.

An analogue computer for induction motor design.— Electr. Energy, 1958, vol. 2, No. 9, p. 362.

Analogue computer predicts radioactive fallout.— Automatic Progr. 1958, vol. 3, No. 7, pp. 254-255, 265.

Au automobile handling simulator.— Process Control and Automat., 1958, September, vol. 5, No. 9, p. 354, 1 illustration.

B. T. C. Analogue Computer.— Engineer, 1958, July 4, vol. 206, No. 5345, p. 30.

Bertein F.— Analogie électrique dans l'étude du traitement d'un lingot par la méthode de la zone fondue.— J. Phys. et Radium 1958, vol. 19, No. 12, Suppl., pp. 182-183.

Cahen F. M. and Carteron J. M.—Analog and digital computers in the French electric power production, transmission, and distribution industry.— Power Appar. and Systems, 1958, February, No. 34, pp. 1533-1538. Bibliography 17.

Corless K. G. and Aldred A. S.— An experimental electronic power system simulator.— Proc. Instn. Electr. Engrs, 1958, October, vol. 105, Pt. A, No. 23, pp. 503-511, 11 illustrations. Bibliography 19.

Designing railway springs by analogue computer.— Process Control and Automat., 1958, September, vol. 5, No. 9, pp. 378-380, 6 illustrations.

Electronic slide rule for packaging calculations.— Packaging 1958, vol. 40, No. 339, p. 64.

Fogel L. J. and Dwonzyk M.— Anticipatory display design through the use of an analog computer.— IRE Wescon Convent. Rec., 1958, vol. 2, Pt. 4, pp. 67-88.

Hálek V.—Zkusenesti z reseni hydraulickych problemu metodom elektroanalogie. (The use of the electrical simulation method for solving hydraulic problems).— Vodni hospodarstvi 1958, No. 10, pp. 297-305.

Hazel W. C.— The use of operational amplifiers in the measurements of transistor parameters.— Review of Sci. Instruments, 1958, vol. 29, No. 3, pp. 235-237.

Horgan J. D. and Pesavento J. A.— Accurate determination of capacitance by conjoining analytical and analog techniques.— Communs. and Electronics, 1958, vol. 77, pp. 397-400, illustrations. Bibliography 7.

Hornfeck A. J. and Imsland T. S.— Performance computer for steam — electric generating units.— Power Apparatus and Systems, 1958, August, No. 37, pp. 647-654, 10 illustrations. Bibliography 10.

Jespers P.— Méthode de calcul analogique de la réponse transistoire des filtres passe — bande symétriques soumis à l'application d'un signal sinusoidal de pulsation quelconque.— Ann. Assoc. Intern. Calcul. Analog., Belg. 1958, September, vol. 1, No. 2, pp. 75-83, 5 illustrations.

Karplus W. J., Bekey G. A. and Pekrul P. I.— Atmospheric diffusion of air pollutants. Analog computer study.— Industr. Engng Chem., 1958, November, vol. 50, No. 11, pp. 1657-1660, 9 illustrations. Bibliography 6.

Kennard L. J. and Nicholson C. H.— Radar simulators.— J. Brit. Inst. Radio Engrs., 1958, January, vol. 18, No. 1, pp. 17-3[illegible], 9 illustrations.

Lonegran J. P.— An introduction to the applications of analogue computers.— Austral. J. Instrum. Technology, 1958, February, vol. 14, No. 1, pp. 62-71, 7 illustrations.

Lowler E. A., Drum L. and Frank V.— Hydraulic problem solution on electronic computers.— J. Waterways and Harbors Div., Proc. Amer. Soc. Civil. Engrs, 1958, vol. 84, No. 1, pp. 1515-1-1515-38.

Meier H. P.— Die Anwendung elektronischer Rechenautomaten in der Maschinen — Industrie.— Schweiz. Bauzeitung, 1958, März 22, Jg. 76, H. 12, SS. 174-176, 3 illustrations.

Meredith J. F. and Freeman E. A.— The simulation of distributed-parameter systems with particular reference to process control problems.— Proc. Instn. Electr. Engrs, 1958, November, vol. 105, Pt. B, pp. 569-576, 13 illustrations.

Moore W. J. M., and Chen C. C.— A study of an analog computing system for an analytical plotter.— Photogrammetria, 1958-1959, vol. 15, No. 1, pp. 14-20.

Nagaraja N. S.— Fourier analysis by a general purpose electronic analogue computer.— J. Instn Telecommun. Engrs., India, 1958, vol. 4, No. 3, pp. 130-136.

Neate R. and Daecy W. J.– A simulation of melting shop operations by means of a computer.– Process control and automation, 1958, July, vol. 5, No. 7, pp. 264-272, 6 illustrations.

Noden D. A.– A balanced precision reference regulator for computer application.–IRE Convention Rec., 1958, vol. 6, Pt. 4, pp. 225-229.

Otley K. O., and Shoemaker R. F.– A voltage-sensitive switch.– Proc. IRE 1958, October, vol. 46, No. 10, pp. 1723-1730, 7 illustrations. Bibliography 36.

Phillips R. A.– Process analysis plus analog simulation yields better mill controls.– Control Engng, 1958, May, vol. 5, No. 5, pp. 113-118, 4 illustrations.

Reethof G.– Application of the analog computer to predict dynamic performance in typical hydraulic circuits.– Trans. ASME, 1958, August, vol. 80, No. 6, pp. 1299-1307, 13 illustrations. Bibliography 14.

Rogers A. E.– The analog computer its operation and use in engineering.– Proc. Instrum. Soc. Amer. 13th Annual Instrum. Automat. Confer. and Exhibit., 1958 Pt. 2.– Pittsburgh, Instrum. Soc. Amer., 1958, Part PCT2, pp. 1-14, 8 illustrations.

Steeg C. W.– Systems engineering and simulation in the military electronics industry.– Proc. Instrum. Soc. Amer. 13th Annual Instrum. Automat. Confer. and Exhibit., 1958 Pt. 2.– Pittsburgh, Instrum. Soc. Amer., 1958, part PCT10, pp. 1-14, 4 illustrations.

Tavoda O.– Zistovanie vztlakov a priesakov pomocu eleptrickej analogie. (Determining pressures and discharges using an electrical analog).–Vodohospod casop. 1958, vol. 6, No. 2, pp. 160-173, Bibliography 10.

Tenger P.– The use of radar simulators in the Royal Navy.– J. Brit. Inst. Radio Engrs 1958, January, vol. 18, No. 1, pp. 33-47, 20 illustrations.

Terrett D. S.– Applications of a general-purpose analogue computer. 2. Analogue computers in chemical engineering.– Chem. and Process Engng 1958, vol. 39, No. 12, pp. 442-445.

Thomas G. E.– Analogue computers in chemical engineering. I. General principles.– Chem. and Process Engng, 1958, vol. 30, No. 10, pp. 343-346.

Wexler H. T. and Stern R. K.– Analog computers in physical simulation.– Proc. Instrum. Soc. Amer., 13th Annual Instrum. Automat. Confer. and Exhibit., 1958, Pt. 2.– Pittsburgh, Instrum. Soc. Amer., 1958, Part PCT8, pp. 1-4, 9 illustrations.

Williams P. J.– Analog computing in the chemical and petroleum industries.– Industr. Engng. Chemistry, 1958, November, vol. 50, No. 11, pp. 1631-1635, illustrations. Bibliography 42.

Woods F. A.– Simulation of process control with an analog computer.– Industr. and Engng Chem., 1958, vol. 50, No. 11, pp. 1627-1630.

Worley Ch. W.– Analog simulation of industrial processes – 1958 Nat. Sympos. Progr. and Trends Chem. and Petrol. Instrum. Wilmington, Del., Pittsburgh Pa, Instrum. Soc. America 1958, pp. 1-17.

G. DIGITAL SIMULATION

IN RUSSIAN

Kozharskii L. A.– On a certain method for designing digital differential analyzers.– In: Problems in the theory of mathematical machines.– Moscow, Fizmatgiz, 1958, pp. 219-230, 6 illustrations. Bibliography 6.

Maiorov F. V.– Digital integrators.– In: Automatic control and digital engineering, No. 1.– Moscow, Mashgiz, 1958, pp. 240-303, 31 illustrations. Bibliography 7.

Zhidkov E. P.– Mathematical fundamentals of a digital differential analyzer.– In: Automatic control and digital engineering, No. 1.– Moscow, Mashgiz, 1958, pp. 228-239, 12 illustrations.

LANGUAGES OTHER THAN RUSSIAN

Beck R. M. and Palevsky M.– The DDA.– Instrum. and Automat., 1958, November, vol. 31, No. 11, pp. 1836-1837, 5 illustrations.

Goldman R. N.– DDA.– What is it ?– Electr. Engng. 1958, July, vol. 77, No. 7, pp. 592-594, 5 illustrations.

Guzman O.– Digital moon-radar antenna programmer with analog rate signal integrator.– IRE Convent. Rec., 1958, vol. 6, Pt. 4, pp. 217-224, 11 illustrations.

Mitchell J. M. and Rugman S.– The Trice – a high speed incremental computer (New parallel digital differential analysers).– IRE, National Convent. Rec., 1958, vol. 6, Pt. 4, pp. 206-209, 9 illustrations.

Rowley G. C.– Digital differential analysers.– Brit. Communs. and Electronics 1958, December, vol. 5, No. 12, pp. 934-938, 6 illustrations. Bibliography 20.

Savastano G.– L'Analizzatore differenziale dell'universita di Napoli.– Electrotecnica, 1958, vol. 45, April, No. 4, pp. 202-212. Lugl., No. 7, pp. 354-365, 15 illustrations. Bibliography 6.

Steward M. H.– Rate limiting in incremental computes.– In: Proc. Computers in Control Systems Conference, 1957.– New York, Amer. Inst., Electr. Engrs, 1958, pp. 87-88.

Superharmonic oscillations as solutions to Duffing's equation as solved by an electronic differential analyzer.— J. Appl. Mech., 1958, June, vol. 25, No. 2, pp. 315-316.

Terao M. and Tamura K.— A semidigital process simulator.— Trans. IRE, 1958, March, vol. 1-7, No. 1, pp. 18-22, 21 illustrations.

Tootill G. C.— The incremental digital computer. (Digital differential analyzer).— Process Control and Automat., 1958, vol. 5, September, No. 9, pp. 350-354, 4 illustrations, October, No. 10, pp. 402-406, 4 illustrations. Bibliography 22.

3. Bibliography

IN RUSSIAN

Vil'dt E. O. and Landsberg R. S.— Bibliography of literature on problems of mathematical simulation (on analog computers) for 1956, Avtomatika i Telemekhanika, 1958, vol. 19, No. 5, pp. 493-506.

No. 10, pp. 402-405, 4 illustrations. Bibliography 32.

V. Bibliography

IN RUSSIAN

Vitt's E. O. and Landsberg R. M. – Bibliography of literature on problems of mathematical simulation (on analog computers) for 1960. – Avtomatika i Telemekhanika, 1962, vol. 10, No. 4, pp. 497-502.

[illegible] mechanisms as solution to [illegible] equation is solved by an electronic differential analyser. – J. Appl. Mech., 1958, [illegible], vol. 25, No. 2, pp. 212-216.

Terao M. and Tsunoda K. – A new digital process simulator. – Trans. IRE, 1959, March, vol. I-7, No. 1, pp. 12-17, 12 illustrations.

Tomlin G. C. – The incremental digital computer. (Digital differential analyser). – Process Control and Automat., 1959, vol. 6, September, No. 9, pp. 351-354, 4 illustrations. [illegible]

[illegible]

Index

AUTOMATION AND REMOTE CONTROL

Volume 21 (1960)

AUTHOR INDEX

1960

Automation and Remote Control

A translation of Avtomatika i Telemekhanika, a publication of the Academy of Sciences of the USSR

Russian Original Dated January, 1959

Vol. 21, No. 1, pp. 1-106 **August, 1960**

CONTENTS

Automation and Remote Control

A translation of Avtomatika i Telemekhanika, a publication of the Academy of Sciences of the USSR

Russian Original Dated February, 1960

Vol. 21, No. 2 October, 1960

CONTENTS

Automation and Remote Control

A translation of Avtomatika i Telemekhanika, a publication of the Academy of Sciences of the USSR

Russian Original Dated March, 1960

Vol. 21, No. 3, pp. 195-302 November, 1960

CONTENTS

Automation and Remote Control

A translation of Avtomatika i Telemekhanika, a publication of the Academy of Sciences of the USSR

Russian Original Dated April, 1960

Vol. 21, No. 4 November, 1960

CONTENTS

[This issue of AVTOMATIKA I TELEMEKHANIKA contained an article commemorating the 90th anniversary of the birth of V. I. Lenin. Since that article did not discuss either the present state of Soviet research in automation or its historic development, and as a matter of fact, contained no information of scientific interest, it was omitted from the translation.]

Automation and Remote Control

A translation of Avtomatika i Telemekhanika, a publication of the Academy of Sciences of the USSR

Russian Original Dated May, 1960

Vol. 21, No. 5, pp. 385-454 December, 1960

CONTENTS

Automation and Remote Control

A translation of Avtomatika i Telemekhanika, a publication of the Academy of Sciences of the USSR

Russian Original Dated June, 1960

Vol. 21, No. 6 **December, 1960**

CONTENTS

CONTENTS (continued)

Automation and Remote Control

A translation of Avtomatika i Telemekhanika, a publication of the Academy of Sciences of the USSR

Vol. 21, No. 7 Russian Original Dated July, 1960 **February, 1961**

CONTENTS

Automation and Remote Control

A translation of Avtomatika i Telemekhanika, a publication of the Academy of Sciences of the USSR

Vol. 21, No. 8 Russian Original Dated August, 1960 March, 1961

CONTENTS

Automation and Remote Control

A translation of Avtomatika i Telemekhanika, a publication of the Academy of Sciences of the USSR

Vol. 21, No. 9 Russian Original Dated September, 1960 April, 1961

CONTENTS

Automation and Remote Control

A translation of Avtomatika i Telemekhanika, a publication of the Academy of Sciences of the USSR

Vol. 21, No. 10 Russian Original Dated October, 1960 **April, 1961**

CONTENTS

Automation and Remote Control

A translation of Avtomatika i Telemekhanika, a publication of the Academy of Sciences of the USSR

Vol. 21, No. 11 Russian Original Dated November, 1960 May, 1961

CONTENTS

Automation and Remote Control

A translation of Avtomatika i Telemekhanika, a publication of the Academy of Sciences of the USSR

Volume 21, No. 12 — Russian Original Dated December, 1960 — July, 1961

CONTENTS

SIGNIFICANCE OF ABBREVIATIONS MOST FREQUENTLY ENCOUNTERED IN SOVIET TECHNICAL PERIODICALS

AN SSSR	*Academy of Sciences, USSR*
FIAN	*Physics Institute, Academy of Sciences USSR*
GITI	*State Scientific and Technical Press*
GITTL	*State Press for Technical and Theoretical Literature*
GOI	*State Optical Institute*
GONTI	*State United Scientific and Technical Press*
Gosénergoizdat	*State Power Press*
Gosfizkhimizdat	*State Physical Chemistry Press*
Goskhimizdat	*State Chemistry Press*
GOST	*All-Union State Standard*
Gostekhizdat	*State Technical Press*
GTTI	*State Technical and Theoretical Press*
IAT	*Institute of Automation and Remote Control*
IF KhI	*Institute of Physical Chemistry Research*
IFP	*Institute of Physical Problems*
IL	*Foreign Literature Press*
IPF	*Institute of Applied Physics*
IPM	*Institute of Applied Mathematics*
IREA	*Institute of Chemical Reagents*
ISN (Izd. Sov. Nauk)	*Soviet Science Press*
IYap	*Institute of Nuclear Studies*
Izd	*Press (publishing house)*
LÉTI	*Leningrad Electrotechnical Institute*
LFTI	*Leningrad Institute of Physics and Technology*
LIM	*Leningrad Institute of Metals*
LITMiO	*Leningrad Institute of Precision Instruments and Optics*
Mashgiz	*State Scientific-Technical Press for Machine Construction Literature*
MGU	*Moscow State University*
Metallurgizdat	*Metallurgy Press*
MOPI	*Moscow Regional Pedagogical Institute*
NIAFIZ	*Scientific Research Association for Physics*
NIFI	*Scientific Research Institute of Physics*
NIIMM	*Scientific Research Institute of Mathematics and Mechanics*
NIKFI	*Scientific Institute of Motion Picture Photography*
NKTM	*People's Commissariat of the Heavy Machinery Industry*
Obrongiz	*State Press of the Defense Industry*
OIYaI	*Joint Institute of Nuclear Studies*
ONTI	*United Scientific and Technical Press*
OTI	*Division of Technical Information*
OTN	*Division of Technical Science*
RIAN	*Radium Institute, Academy of Sciences of the USSR*
SPB	*All-Union Special Planning Office*
Stroiizdat	*Construction Press*
URALFTI	*Ural Institute of Physics and Technology*
TsNIITMASh	*Central Scientific Research Institute of Technology and Machinery*
VNIIM	*All-Union Scientific Research Institute of Metrology*

NOTE: Abbreviations not on this list and not explained in the translation have been transliterated, no further information about their significance being available to us — *Publisher.*

Publication of a "Soviet Instrumentation and Control Translation Series" by the Instrument Society of America has been made possible by a grant in aid from the National Science Foundation, with additional assistance from the National Bureau of Standards for the journal *Measurement Techniques.*

Subscription rates have been set at modest levels to permit widest possible distribution of these translated journals.

The Series now includes four important Soviet instrumentation and control journals. The journals included in the Series, and the subscription rates for the translations, are as follows:

MEASUREMENT TECHNIQUES – *Izmeritel'naya Tekhnika*

Russian original published by the Committee of Standards, Measures, and Measuring Instruments of the Council of Ministers, USSR. The articles in this journal are of interest to all who are engaged in the study and application of fundamental measurements. Both 1958 (bimonthly) and 1959-1960 (monthlies) available.

Per year (12 issues) starting with 1960, No. 1
General: United States and Canada . . $25.00
Elsewhere 28.00
Libraries of nonprofit academic institutions:
United States and Canada . . $12.50
Elsewhere 15.50

INSTRUMENTS AND EXPERIMENTAL TECHNIQUES
Pribory i Tekhnika Éksperimenta

Russian original published by the Academy of Sciences, USSR. The articles in this journal relate to the function, construction, application, and operation of instruments in various fields of experimentation. 1958, 1959, and 1960 issues available.

Per year (6 issues) starting with 1960, No. 1
General: United States and Canada . . $25.00
Elsewhere 28.00
Libraries of nonprofit academic institutions:
United States and Canada . . $12.50
Elsewhere 15.50

AUTOMATION AND REMOTE CONTROL – *Avtomatika i Telemekhanika*

Russian original published by the Institute of Automation and Remote Control of the Academy of Sciences, USSR. The articles are concerned with analysis of all phases of automatic control theory and techniques. 1957, 1958, 1959, and 1960 issues available.

Per year (12 issues) starting with Vol. 21, No.1
General: United States and Canada . . $35.00
Elsewhere 38.00
Libraries of nonprofit academic institutions:
United States and Canada . . $17.50
Elsewhere 20.50

INDUSTRIAL LABORATORY – *Zavodskaya Laboratoriya*

Russian original published by the Ministry of Light Metals, USSR. The articles in this journal relate to instrumentation for analytical chemistry and to physical and mechanical methods of materials research and testing. 1958, 1959, and 1960 issues available.

Per year (12 issues) starting with Vol. 26, No. 1
General: United States and Canada . . $35.00
Elsewhere 38.00
Libraries of nonprofit academic institutions:
United States and Canada . . $17.50
Elsewhere 20.50

Single issues of all four journals, to everyone, each . . . $6.00

Prices on 1957, 1958, and 1959 issues available upon request

SPECIAL SUBSCRIPTION OFFER:

One year's subscription to all four journals of the 1960 Series, as above listed:

General: United States and Canada . . $110.00
Elsewhere 122.00
Libraries of nonprofit academic institutions:
United States and Canada . . $ 55.00
Elsewhere 67.00

Subscriptions should be addressed to:

Instrument Society of America
530 William Penn Place
Pittsburgh 19, Penna.

#701